MIGRACIÓN KIRCHHOFF EMPLEANDO DATOS COMPRIMIDOS MEDIANTE UNA DESCOMPOSICIÓN MATCHING PURSUIT

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A Luz Marina y Edgar Iván: ustedes son mi mayor fortaleza.

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RESUMEN

TÍTULO: Migración Kirchhoff empleando datos comprimidos mediante una descomposición Matching Pursuit*

AUTOR: Edher Fabián Sánchez Colmenares**

PALABRAS CLAVE: Compresión de datos sísmicos, Algoritmos Matching Pursuit, Migración Kirchhoff, Matlab.

DESCRIPCIÓN:

La migración Kirchhoff es uno de los métodos estándar utilizados por la industria petrolera para procesar datos sísmicos. Este método se basa en mapear de tiempo a profundidad las muestras de los datos de entrada de acuerdo con las tablas de tiempo de viaje. El objetivo de la migración es obtener una imagen sísmica de mayor precisión. Actualmente, la cantidad de datos a procesar puede ser del orden de los Terabytes, lo cual demanda altos costos de almacenamiento y cómputo. Por lo tanto, una alternativa para llevar a cabo de manera eficiente el proceso de migración es emplear técnicas de compresión de datos sísmicos para reducir el tamaño de los datos a procesar y desarrollar el proceso de migración en un dominio comprimido.

Este proyecto de investigación desarrolla una migración 2D Kirchhoff pre-apilada sobre datos sísmicos comprimidos mediante algoritmos Matching Pursuit. Se utilizaron tres diferentes algoritmos Matching Pursuit (MP, OMP y OLS) y se plantearon dos estrategias de migración. Se utilizó la Relación Señal-Ruido (SNR), el error de amplitud en los reflectores y el espectro de Fourier como métricas para determinar la calidad de los resultados obtenidos por el método propuesto. Las pruebas se realizaron sobre tres modelos de velocidades sintéticos y se tomó el factor de compresión (CR) como variable independiente.

Los resultados muestran que el método OLS ofrece una SNR por encima de 40 [dB], con un error de amplitud inferior al 0,1 % en los reflectores para un CR de 10. Para un CR de 20, se obtuvo un error del 0.1 % en términos de magnitud en el espectro de frecuencia. Por otro lado, OMP ofrece los mejores resultados en términos de calidad/tiempo de ejecución, ya que requiere menos esfuerzos computacionales que OLS. Finalmente, los resultados sugieren la posibilidad de realizar la migración Kirchhoff comprimiendo los datos sísmicos hasta 20:1 sin afectar significativamente los atributos sísmicos de la imagen.

* Trabajo de investigación.

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ABSTRACT

TITLE: Kirchhoff Migration using compressed data by a Matching Pursuit decomposition.*

AUTHOR: Edher Fabián Sánchez Colmenares**

KEYWORDS: Seismic data compression, Matching Pursuit algorithms, Kirchhoff migration, Matlab.

DESCRIPTION:

Kirchhoff migration is one of the standard methods used by the oil industry to process seismic data. This method is based on mapping from time to depth the input data samples according to the travel time tables. The objective of the migration is to obtain a more accurate seismic image. Currently, the amount of data to be processed can be in the order of Terabytes, which demands high storage and computational costs. Therefore, an alternative to efficiently carry out the migration process could be to employ seismic data compression techniques to reduce the size of the data to be processed and to develop the migration process in a compressed domain.

This research project develops a pre-stacked 2D Kirchhoff migration on compressed seismic data using Matching Pursuit algorithms. Three different Matching Pursuit algorithms were used (MP, OMP and OLS) and two migration strategies were proposed. The Signal-to-Noise Ratio (SNR), the amplitude error in the reflectors and the Fourier spectrum were used as metrics to determine the quality of the results obtained by the proposed method. The tests were performed on three synthetic velocity models and the compression ratio (CR) was taken as independent variable.

The results show that the OLS method offers an SNR above of 40 [dB], with an amplitude error in the reflectors less than 0.1 % for a CR of 10. For a CR of 20, it was obtained an error of 0.1 % in terms of magnitude in the frequency spectrum. On the other hand, OMP offers the best results in terms of quality/execution time, since it requires less computational effort than OLS. Finally, the results suggest the possibility of performing the Kirchhoff migration by compressing the seismic data up to 20:1 without significantly affecting the seismic attributes of the image.

* Research work.

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INTRODUCTION

The present project is framed within the research program "Pre-stacked seismic migration in depth by extrapolation of wave fields using high performance computing for massive data in complex zones" sponsored by COLCIENCIAS-Ecopetrol in association with the Industrial University of Santander (UIS). This research aims to increase the resolution of the subsurface images required by the oil industry to identify possible hydrocarbon reserves. Also, this research seeks to reduce the execution time of seismic applications by using computer architectures different from CPUs.

Nowadays there are different methods of seismic exploration which are based on the generation of seismic waves from an artificial source. The waves are propagated through the subsurface and then, collected by a series of sensors called geophones. Once this process is done, several seismic data processing stages are performed to obtain an image that approximately represents the structure and composition of the subsurface. These stages include: filtering, deconvolution, common mid-point sorting (CMP), velocity analysis, normal move-out correction (NMO), stacking and migration [1].

The migration process is one of the most computationally expensive. This module is responsible for relocating the reflectors to their true position and collapse the diffraction, in order to obtain a subsurface image closer to the real life [2]. There are different methods to migrate seismic data, all of them based on the solution of the wave equation or applying principles of optical physics. One of the common methods that have been employed by the oil industry for several decades is the Kirchhoff migration. The algorithm starts at some point of interest from the image and uses the travel times tables to determine which samples, of the collected seismic data, have contribution on that point. The main advantages of this method are its speed and flexibility [1].

The amount of data to be processed is related with the structure complexity that presents the area of interest. Currently, the seismic data to process can be in the order of hundreds Terabytes [3] [4] [5]. Dealing with this amount of data strongly affects the performance of the migration process in storage and computational terms. Therefore, a possible solution is to use compression algorithms, which allow representing the same information with a smaller size of data than the original. In general, compression algorithms consist of three stages: transformation, quantization and coding [3] [6]. Nevertheless, as the coding represents the data into code-words, it is not reasonable to perform operations on them, implying the need of uncompressing the data to develop the respective process. From that premise, this work seeks to use a seismic data compression strategy that allows performing the Kirchhoff migration in the compressed domain without significantly affecting the seismic attributes.

Related works:

There have been several works on the Kirchhoff migration using wavelets to process the seismic data in a compressed domain. In [7], the samples that represent local extrema in a window of length equivalent to the seismic wavelet, are added to a vector that will be migrated. The selection of samples is made by a sorting procedure. A reconstruction process is developed after migration by convolving the seismic section with wavelets. In [8], the seismic traces are compressed into coefficients by a wavelet decomposition, and then a 2D Kirchhoff migration is applied according to their time location. In [9], a 3D Kirchhoff migration on compressed real data is developed using a similar strategy as in [8]. A common factor in these works is a post-migration process to improve the quality of the obtained image.

A different approach to perform the Kirchhoff migration on compressed data is through the Matching Pursuit decomposition (MP). Given a dictionary $\Phi \in R^{N \times M}$ of redundant functions called atoms, it is possible to approximate a signal $f \in R^N$ by a linear combination of k elements from a dictionary, with $k << M$ [10]. In [11], the seismic traces are decomposed by MP using the Ricker wavelets as atoms and then, the migration process is performed atom per atom obtaining considerable compression ratio. In [12], an estimated wavelet is used to compute the atoms of the dictionary, in order to avoid noise sensitivity. The time and amplitude of the atoms selected are called Matching Pursuit coefficients (MPCs), which are migrated as samples of the original data.

In this work, we proposed to compress the seismic data into coefficients by three different Matching Pursuit algorithms: Matching Pursuit (MP), Orthogonal Matching Pursuit (OMP) and Orthogonal Least Square (OLS). These algorithms have been applied in different seismic applications such as decomposition [13] [14] and denoising process [15]. For each method, the Ricker wavelet was selected as atom to decompose the seismic data [16]. Then, the coefficients were used through a linear combination process to compute the amplitude contributions required by the travel time tables directly in the image. The Ricker waveform equation and some parameters from the compression stage were introduced into the Kirchhoff operator to transform from the Ricker domain to the travel time tables domain during the migration process.

The principal advantage of this strategy lies in changing most of the memory access operations for mathematical operations (sums, substractions, multiplications, etc..) More specifically, we aim to avoid as much as possible the bottleneck produced by extracting information from the disk, by adding computational operations which are less expensive. The proposed method was tested using 2D synthetic data, obtaining considerable compression ratio (up to 20:1) without significantly affecting some attributes of the image such as frequency spectrum and reflectors amplitude.

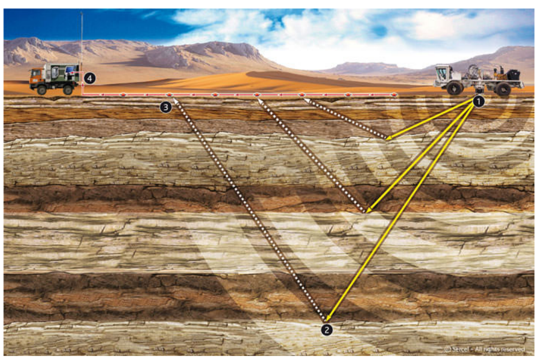
The present document is organized as follows: Chapter 1 presents a brief explanation of the exploration and migration process. Chapter 2 shows the three Matching

Pursuit algorithms employed to compress the seismic data. Chapters 3 presents the proposed strategies to develop the Kirchhoff migration on compressed seismic data. The results are analyzed in Chapter 4 and finally, Chapter 5 presents the conclusions and recommendations.

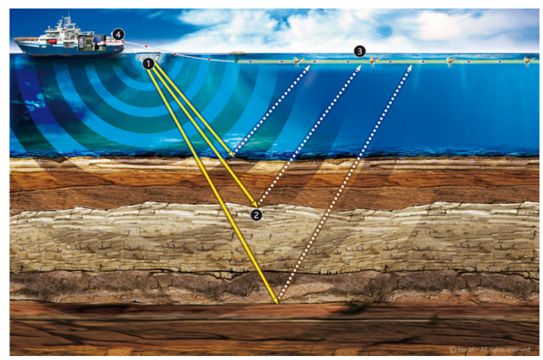
1. SEISMIC EXPLORATION: FUNDAMENTALS

Geophysics is the science responsible for studying the structure and composition of the Earth and the physical agents that modify it. These studies are fundamental in the seismic explorations carried out by the oil industry since they help to detect the presence or not of hydrocarbons in the subsurface. One of the techniques most used to collect geological information is the seismic reflection (figure 1).

Figure 1: Seismic data acquisition in a) land and b) sea [17].



(a)

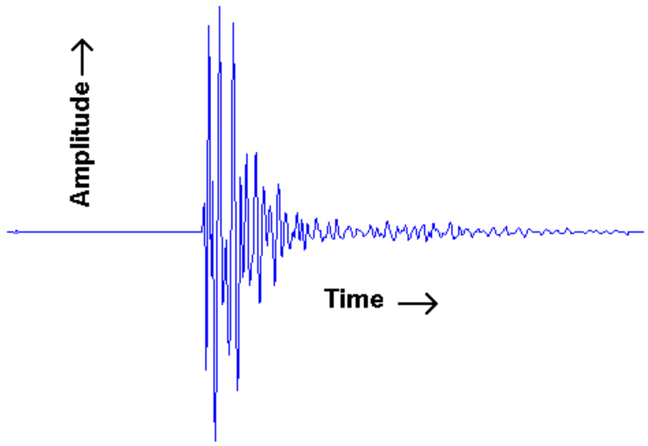


(b)

This method is based on employing different artificial sources such as vibrators or explosives to propagate signals through the layers of the subsurface (also called reflectors). The reflected signals are recorded by a series of geophones laid out in the subsurface. The signal collected by each geophone is called seismic trace and the group of seismic traces collected from one shot is defined as shot gather. Figure 2 shows an example of a seismic trace.

After performing the acquisition stage, it is necessary to develop the seismic data processing stage. These stages consists of: filtering, deconvolution, common mid-point sorting (CMP), velocity analysis, normal move-out correction (NMO), stacking and migration [1]. The migration process is the main focus in this research work.

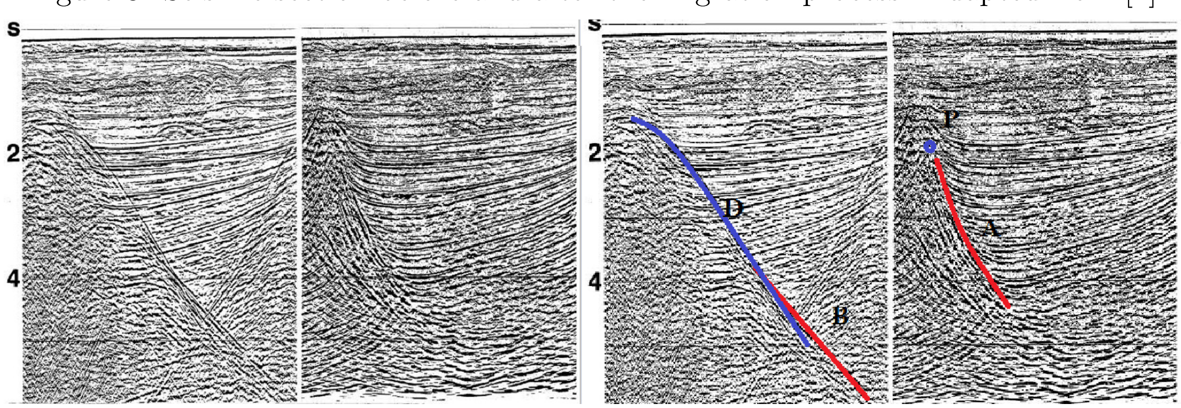
Figure 2: Standard seismic trace [18].



1.1 Seismic Migration

This stage is the last module in the seismic data processing. Seismic migration is a term used in the seismic reflection to describe the process of relocating the recorded events (seismic traces) to their correct spatial position and collapses the diffraction energy to their scattering points [1]. An alternative definition is to consider the migration as an inverse process, where the recorded events are propagated back to its associated reflector locations [2]. Figure 3 presents an example where the reflector B is relocated to its "real" position A and the diffraction D is collapsed to P.

Figure 3: Seismic section before and after the migration process. Adapted from [1]



Up to the 1960s, the migration process was developed by graphics methods [19]. Nowadays, there are different numerical methods to perform the migration process on seismic data. These methods can be classified as follow [1]:

- Based on integral methods: Kirchhoff migration.
- Based on finite differences: Reverse time migration (RTM).
- Based on transformation: *f-k* migration.

In the same way, the seismic migration can be applied in different forms, also called types [5]:

- Post or pre-stack migration: If the migration process is performed after stacking the seismic data it is called post-stack migration, otherwise, it is pre-stack migration. The post-stack migration saves computational efforts in comparison to the pre-stack migration but, the quality of the results is lower.
- 2D or 3D Migration: It depends on the geometry of the acquisition. If the geophones are located linearly the migration is performed in 2D but, if the geophones occupy a determined area it is suitable to perform a 3D migration.
- Time or depth migration: It is related to the velocity model. If the velocity increases as the time pass and the horizontal variations are gradual it is applied a time migration. Meanwhile, if the velocity model has strong changes in both vertical and horizontal directions, it is applied a depth migration.

1.2 Kirchhoff Migration

As mentioned, this method is derived from the solution of the acoustic wave equation given by:

$$\frac{\partial^2 P(x,z,t)}{\partial x^2} + \frac{\partial^2 P(x,z,t)}{\partial z^2} = \frac{1}{c^2(x,z)} \frac{\partial^2 P(x,z,t)}{\partial t^2} + S(x,z,t), \tag{1.1}$$

where $P(x,z,t)$ represents the wave's pressure intensity, $c(x,z)$ is the velocity of the medium, $S(x,z,t)$ is the source and, x and z represent a location in space in terms of distance and depth respectively. The solution of the equation is performed by applying Green's function [20] and it can be described in discrete form by [21]:

$$M(x,z) = \sum_{k=1}^{ns} \sum_{j=1}^{nr} w(r_j, s_k, x, z, tt) I(r_j, s_k, tt), \tag{1.2}$$

where $M(x,z)$ is the migrated image in depth and $I(r_j, s_k, tt)$ represents the samples of the input data required by the travel time tables tt. This process is closely related to diffraction summation but with an addition weight function w that consists in three factors: obliquity factor, spherical spreading factor and wavelet shaping factor [1].

It should be mentioned that, to perform the Kirchhoff migration it is necessary to compute the travel time tables first on the given velocity model. The numerical solution of the Eikonal equation by finite differences is one of the methods used to perform travel time [22]. The Eikonal equation is defined as:

$$\left(\frac{\partial t}{\partial x}\right)^2 + \left(\frac{\partial t}{\partial z}\right)^2 = s(x,z)^2, \tag{1.3}$$

where $s(x,z)$ is the slowness or reciprocal of velocity. The points of the travel time tables have a value that represents the time spent by the wave to get there from a specific source. According to those points, the samples of the input data are mapped in depth to obtain the migrated image.

2. MATCHING PURSUIT ALGORITHMS

In this chapter, we present the three algorithms used to compress the seismic data that will be used later as input data for the Kirchhoff migration. A brief mathematical description of the three algorithms is presented and the results of the simulations on synthetic data are shown.

2.1 Matching Pursuit

Matching Pursuit (MP) is a greedy algorithm that allows the decomposition of a signal into a linear expansion of waveforms, which are chosen from a redundant dictionary of functions Φ [10]. The waveforms are discrete-time signals called atoms and denoted by $g_\gamma(t)$, such that $\|g_\gamma(t)\|_2 = 1$, where γ represents operations of scaling, frequency modulation and translations. These atoms are defined as:

$$g_\gamma(t) = \frac{1}{\sqrt{s}} g\left(\frac{t-u}{s}\right) e^{i\xi t} \tag{2.1}$$

The algorithm selects in each iteration the waveforms that are best correlated to the signal by a succession of orthogonal projections of $f(t)$ on the dictionary. Once an atom is selected ($g_{\gamma 0}$), the signal can be decomposed as:

$$f = \langle f, g_{\gamma 0} \rangle g_{\gamma 0} + Rf, \tag{2.2}$$

where Rf is the residual after approximating f on $g_{\gamma 0} \in \Phi$. In each step, $\|Rf\|_2$ is minimized by choosing a g_{γ_i} such that $|\langle Rf, g_{\gamma_i} \rangle|$ is maximum. Assuming $R^0 f = f$, the algorithm iteratively decompose the calculated residuals until a stopping criterion. Thus the signal can be expressed as:

$$f = \sum_{i=0}^{m-1} g_{\gamma i} \langle R^i f, g_{\gamma i} \rangle + R^m f = \Phi_{\Gamma^m} \alpha^m + R^m f, \tag{2.3}$$

where Φ_{Γ^m} represents the columns selected from the dictionary, Γ^m is the set of indices of the selected columns and α^m is the solution vector. The MP process can be summarized as:

Algorithm 1 Matching Pursuit (MP)

Input: $f = R^0 f$, $\alpha^0 = 0$ and $\Gamma^0 = \emptyset$.
Output: Γ^k, α^k and f^k.

```
for k = 1 : stopping criterion is met do
    Compute the correlation: z^k = Φ^T R^{k-1} f
    Find a g_{γi} such that: i = arg_j max|z^k(j)|
    Update: Γ^k = Γ^{k-1} ∪ i
    Update: α^k = z^k(i)
    Compute new residue: R^k f = f − Φ_{Γ^k} α^k
end for
```

2.2 Orthogonal Matching Pursuit

Orthogonal Matching Pursuit (OMP) was developed as an improvement of the Matching Pursuit algorithm [23] [24]. The OMP method uses the same criterion as MP to select the atoms from the dictionary but, the process to compute the solution vector is different. In each iteration, the selected atoms are used to find a set of coefficients that minimize the distance to f. This process can be described as follows:

$$f \approx \Phi_{\Gamma^k} \Phi_{\Gamma^k}^{\dagger} f, \tag{2.4}$$

where $\Phi_{\Gamma^k}^{\dagger}$ is the pseudoinverse of Φ_{Γ^k} defined as:

$$\Phi_{\Gamma^k}^{\dagger} = \left(\Phi_{\Gamma^k}^{*} \Phi_{\Gamma^k}\right)^{-1} \Phi_{\Gamma^k}^{*}, \tag{2.5}$$

where * represents the conjugate transpose. The process of finding the closest approximation to f in each step, gives a faster convergence to the optimal solution in comparison to MP. Nevertheless, it is important to mention that the OMP method requires more computational efforts because of the addition of matrix operations to find the solution vector. The OMP process can be summarized as:

Algorithm 2 Orthogonal Matching Pursuit

Input: $f = R^0 f$, $\alpha^0 = 0$ and $\Gamma^0 = \emptyset$.
Output: Γ^k, α^k and f^k.

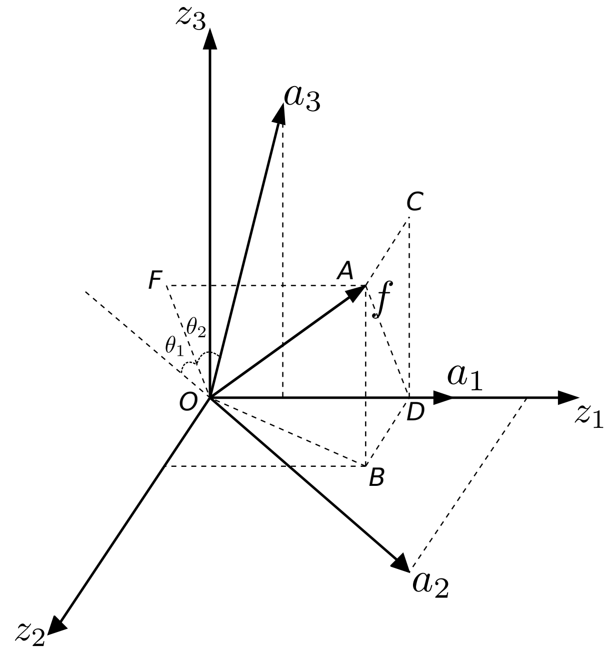
```
for k = 1 : stopping criterion is met do
    Compute the correlation: z^k = Φ^T R^{k-1} f
    Find a g_{γi} such that: i = arg_j max|z^k(j)|
    Update: Γ^k = Γ^{k-1} ∪ i
    Solve least square problem: α^k = Φ^†_{Γ^k} f
    Compute new residue: R^k f = f − Φ_{Γ^k} α^k
end for
```

2.3 Orthogonal Least Squares

Before the mathematical description of the Orthogonal Least Squares algorithm (OLS), it is important to mention that this process was called with other names in the literature as an improvement of OMP [25], [26] and [27]. However, in [28] and [29] the authors clarified the confusion and explained in detail both methods with their differences.

The OLS method computes the solution vector as OMP but, the selection of the atom is different than OMP and MP. In each iteration, OLS search the atom $g_{\gamma i}$ that gives the minimum least square error of the residue taking into account the previously atoms selected. In figure 4 is presented an example to illustrate the difference between OMP and OLS.

Figure 4: Illustrating OMP vs OLS (Adapted from [30]).



Denote $\overrightarrow{a_1}, \overrightarrow{a_2}$ and $\overrightarrow{a_3}$ as the possible atoms to choose from the dictionary and $\overrightarrow{f}$ the signal of interest. Assume $\overrightarrow{a_1}$ to be overlapped on z_1, $\overrightarrow{a_2} \in z_1 z_2$ plane and $\overrightarrow{a_3} \in z_1 z_3$ plane. Initially, suppose that $\overrightarrow{a_1}$ is the most correlated to $\overrightarrow{f}$ for both OMP and OLS, therefore the residue is $\overrightarrow{AD} = \overrightarrow{OF}$. Now, in the case of OMP, the second atom is selected based on the inner product, i.e, the angles between the residual and the remaining atoms. If we define θ_1 and θ_2 as the angles between $\overrightarrow{OF} - \overrightarrow{a_2}$ and $\overrightarrow{OF} - \overrightarrow{a_3}$ respectively, OMP selects $\overrightarrow{a_2}$ because of $\theta_1 < \theta_2$. On the other hand, OLS selects the second atom based on the norm of the residues. If we define $\overrightarrow{AB}$ and $\overrightarrow{AC}$ as the residues obtained by projecting f into the $\overrightarrow{a_1} - \overrightarrow{a_2}$ plane and $\overrightarrow{a_1} - \overrightarrow{a_3}$ plane respectively, OLS selects $\overrightarrow{a_3}$ since $\|\overrightarrow{AC}\|_2 < \|\overrightarrow{AB}\|_2$.

Thus, the selection process of OLS can be described as:

$$i = arg_j min \left\| f - \Phi_{\Gamma^k_j} \Phi^{\dagger}_{\Gamma^k_j} f \right\|_2, \qquad j \notin \Gamma^{k-1}, \tag{2.6}$$

where i represents the index of the atom selected and Γ^{k-1} is the set of index selected during the process. Note that the OLS process is indeed more expensive than OMP due to it is necessary to perform as many matrix inversions as atoms are not used in each step, while OMP just does it once. The OLS process can be summarized as follows:

Algorithm 3 Orthogonal Least Square

Input: $f = R^0 f$, $\alpha^0 = 0$ and $\Gamma^0 = \emptyset$.
Output: Γ^k, α^k and f^k.

for $k = 1$: stopping criterion is met **do**
    Compute minimum error: $i = arg_j min_{\Gamma^k : \Gamma^k = \Gamma^{k-1} \cup j} \left\| R^k f \right\|_2$.
    Update: $\Gamma^k = \Gamma^{k-1} \cup i$
    Solve least square problem: $\alpha^k = \Phi^{\dagger}_{\Gamma^k} f$
    Compute new residue: $R^k f = f - \Phi_{\Gamma^k} \alpha^k$
end for

2.4 Decomposing seismic data by MP algorithms

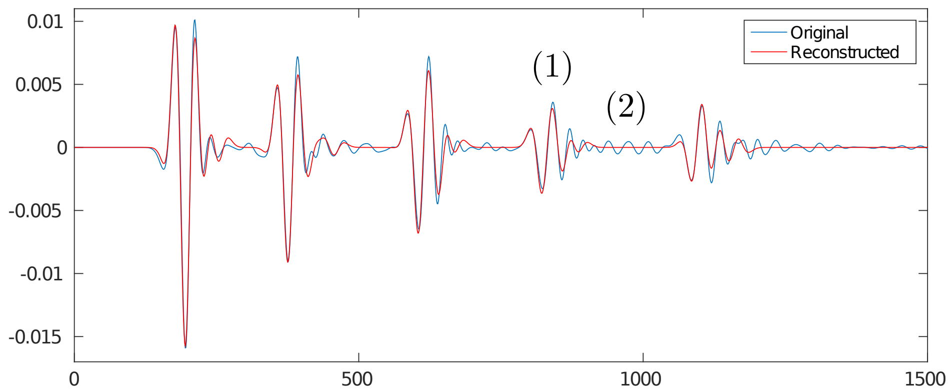
In this section, the three Matching Pursuit algorithms are applied on synthetic seismic data to test their responses. The Signal-Noise Ratio (SNR) between the original and the reconstructed data was employed as metric to estimate the quality of the results. The stopping criterion of the processes was the amount of atoms used to decompose the seismic data. The Ricker waveform was selected as atom to decompose the seismic data, which is defined as:

$$R(t) = (1 - 2(t - t_p)^2 f^2 \pi^2) e^{-(t-t_p)^2 f^2 \pi^2}, \tag{2.7}$$

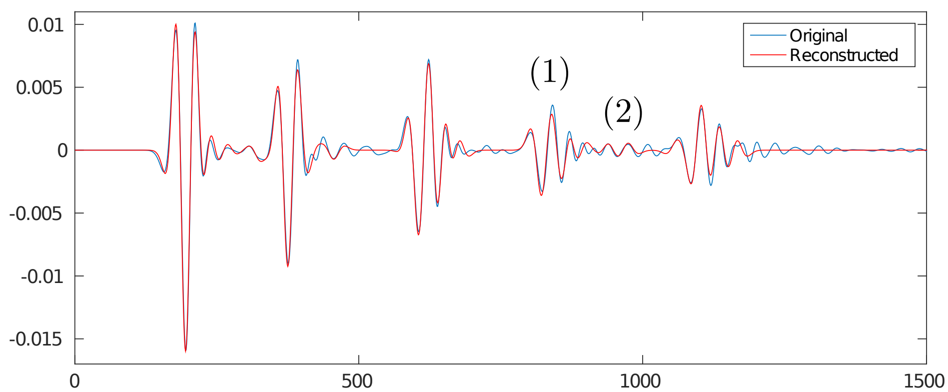
where f is the frequency of the signal and t_p is the delay of maximum peak. To create the dictionary, we applied shifting operations to the atoms.

Figure 5 shows the responses of the three Matching pursuit algorithms applied on a seismic trace. Note that the OLS process reconstructed better the original amplitudes from the seismic trace than the other methods (see (1) in figure 5). This can be explained because OLS focus on reducing as much as possible, the difference in amplitude between the atoms selected and the signal during the selection process. On the other hand, OMP reconstructed more parts from the original signal than the other methods (see (2) in figure 5). Unlike OLS, OMP selects the atoms by prioritizing the difference in phase between the signal and the atoms. In the case of MP, although the selection process is the same as OMP, the fact that OMP finds a better solution vector from the

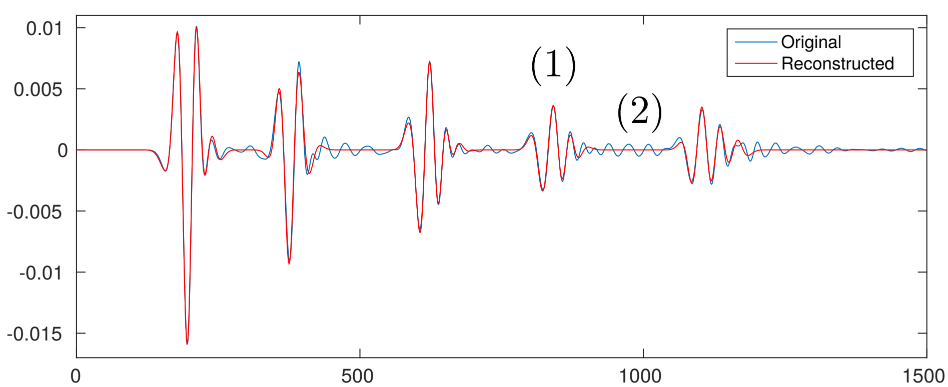
Figure 5: Decomposition process on a seismic trace by the three Matching Pursuit algorithms using 29 atoms. The SNR between the original and reconstructed signal are a) 14.3 [dB], b) 16.7 [dB] and c) 17.7 [dB] for MP, OMP and OLS respectively.



(a) Decomposition by MP.



(b) Decomposition by OMP.

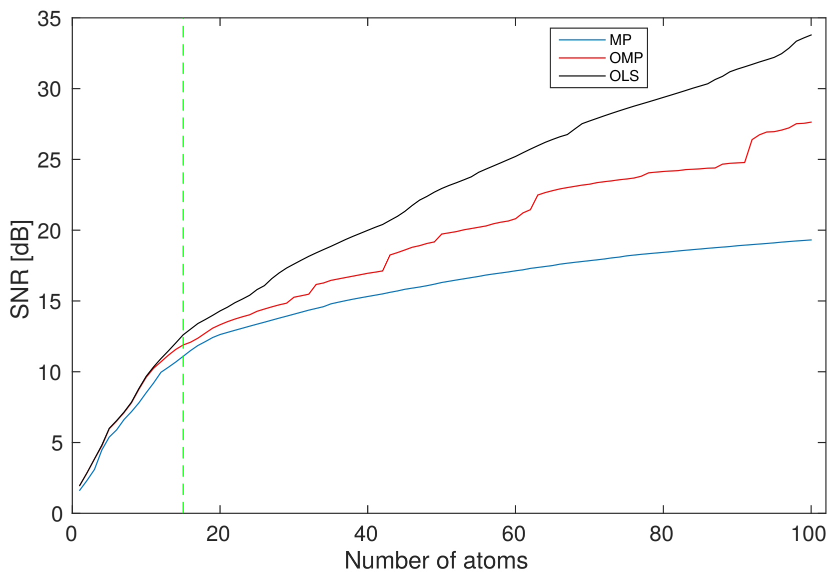


(c) Decomposition by OLS.

atoms selected, it gives the advantage of reconstructing the same sections of the signal with fewer atoms.

Figure 6 presents the relationship between the SNR and the number of atoms employed by the three Matching Pursuit algorithms. Note that the three algorithms achieved similar results until 15 atoms (green line). From there, the differences between the results in terms of SNR start to be significant. Moreover, it is possible to observe that the growth rate decreased from that point. This could be explained because the greatest amplitudes of the seismic trace were reconstructed in that point, i.e, with 15 atoms it was possible to represent the most relevant parts of the signal in terms of amplitude.

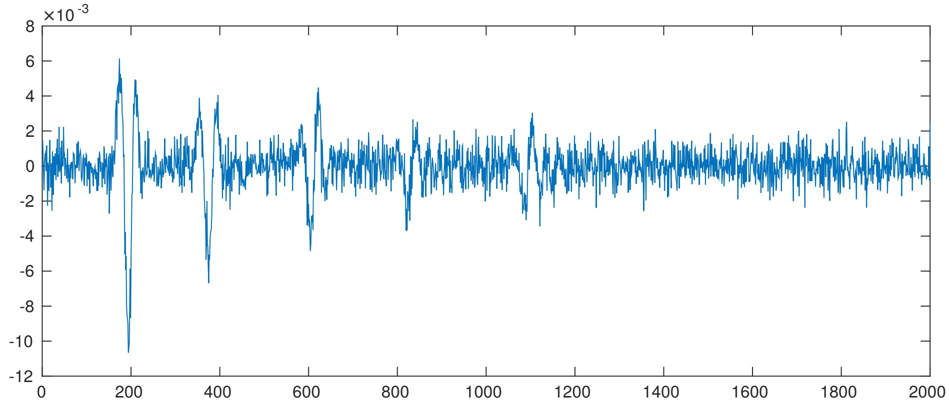
Figure 6: The SNR between the original and reconstructed seismic trace by using the three MP algorithms for different amount of atoms.



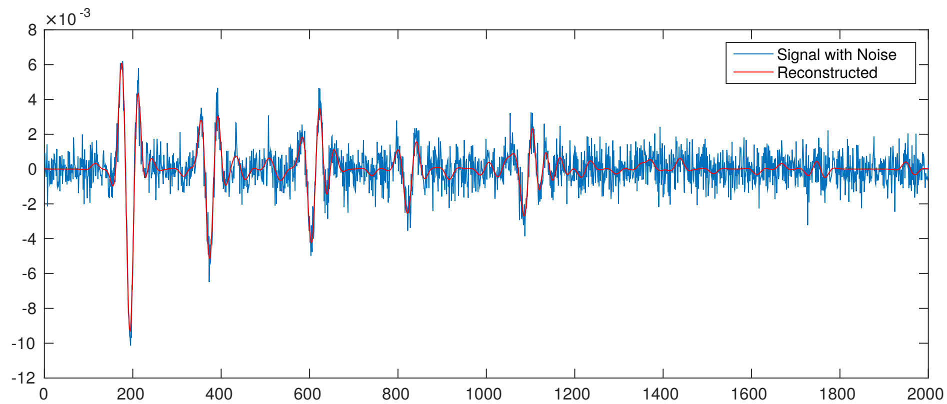
In the same way, the three Matching Pursuit algorithms were tested on a seismic trace with noise in order to analyze its responses. Additive white Gaussian noise (AWGN) was applied on the original seismic trace from figure 5 with a 3 [dB] of SNR. Figure 7a shows the signal with noise and figure 7b presents the reconstructed signal by applying the OMP process using 57 atoms (red line). The reconstructed signal is compared with the original signal in terms of SNR, obtaining 13.6 [dB] (figure 7c). Note that the OMP decomposition worked as a denoising process, keeping most of the relevant amplitudes and filtering the noise.

Figure 8 shows the responses achieved by applying the three Matching Pursuit algorithms on the seismic trace from figure 7a using differents amount of atoms. The reconstructed traces were compared against the original trace in terms of SNR. The OMP and OLS responses had similar behavior, rapidly increasing to approximately 20 atoms (green line) and from there, OMP maintained almost a stable response while

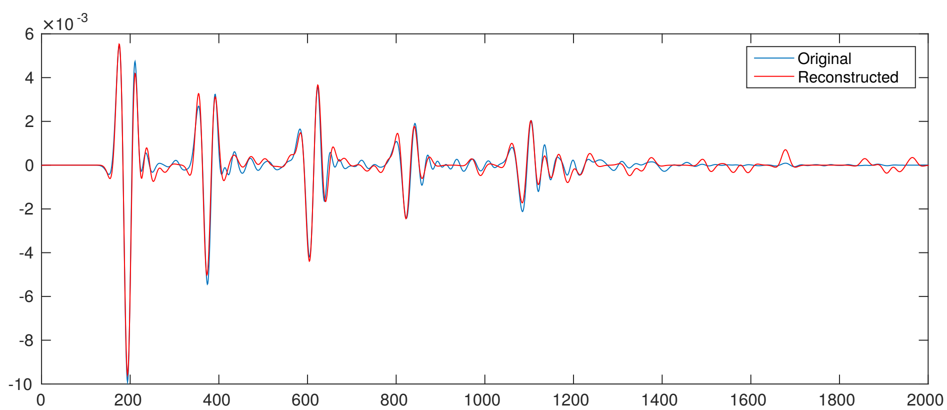
Figure 7: OMP process on a seismic trace with 3 [dB] AWGN using 57 atoms.



(a) Seismic trace with noise.



(b) Decomposition by OMP.



(c) Original trace vs reconstructed trace.

OLS decreased about 2 [dB]. In the case of MP, the results in terms of SNR were lower than the other methods up to 110 atoms.

Figure 8: The SNR obtained by applying the three MP algorithms on the seismic trace with noise for different amount of atoms.

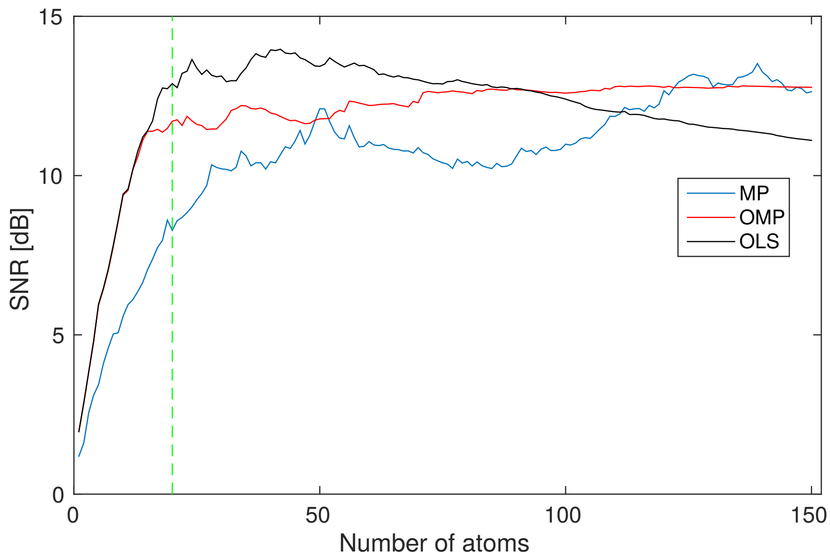


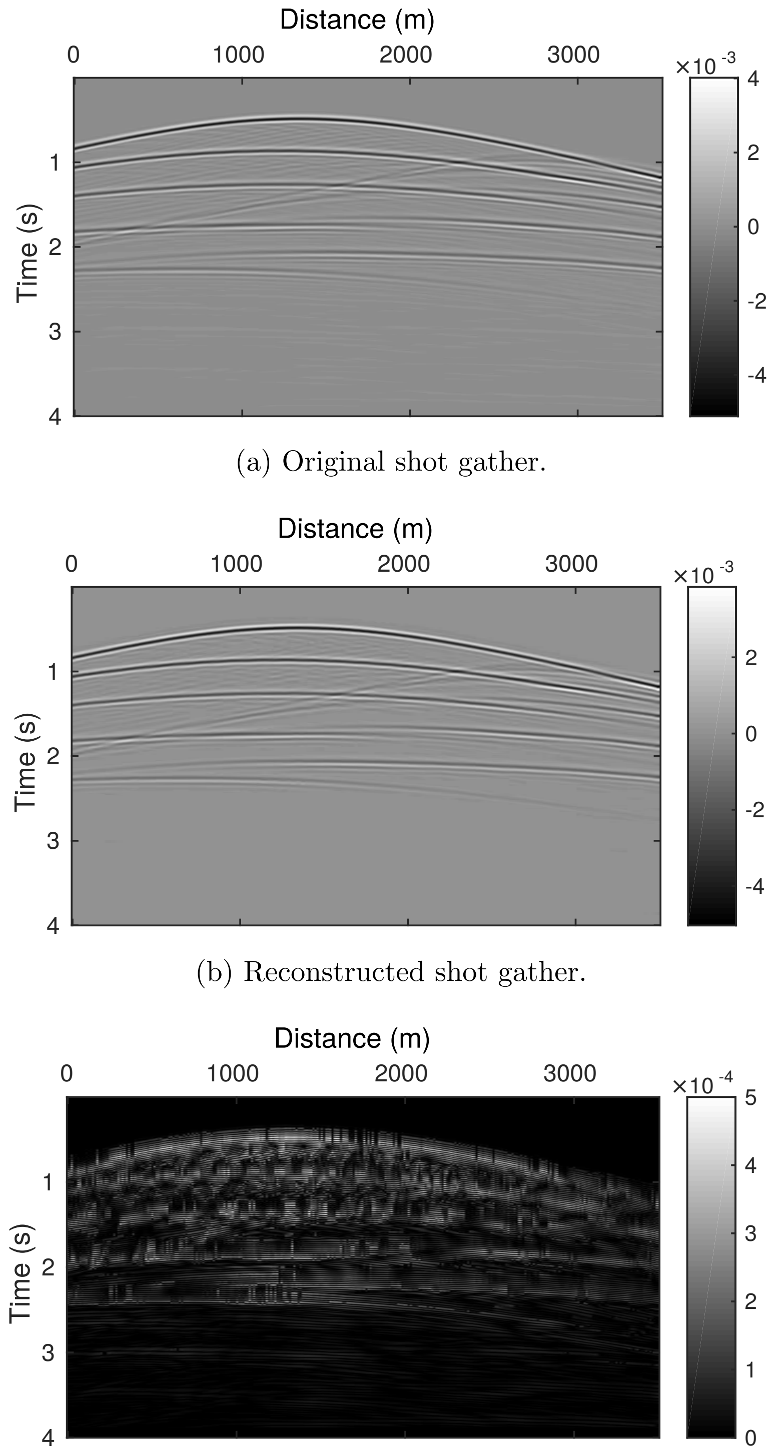
Table 1 shows the best results obtained by the MP algorithms in terms of SNR for different values of noise. Based on the location of the stable responses from figure 8, it was taken 50 atoms as stopping criterion of the algorithms. The SNR values were obtained by developing 50 experiments for each MP algorithm and then compute the mean value of the maximum values obtained from each experiment.

Table 1: Mean value of the SNR after applying the MP algorithms to a seismic trace with differents noise values.

Tested noise values	MP	OMP	OLS
3 [dB]	6.2 [dB]	13.2 [dB]	14.9 [dB]
1 [dB]	3.1 [dB]	12.1 [dB]	13.3 [dB]
-3 [dB]	-1.2 [dB]	10 [dB]	10.4 [dB]

The next step was to use a shot gather to test the three Matching Pursuit algorithms. Figure 9a shows the shot gather and figure 9b presents the reconstructed shot gather using the OMP decomposition. It was used 57 atoms to decompose each seismic trace, yielding an SNR of 17.47 [dB] between the seismic sections. The residuals between the sections are shown in figure 9c, bounded with a maximum error of 5×10^{-5}. The error was calculated by the magnitude of the differences between the original and the reconstructed shot gather.

Figure 9: OMP process applied on shot gather.



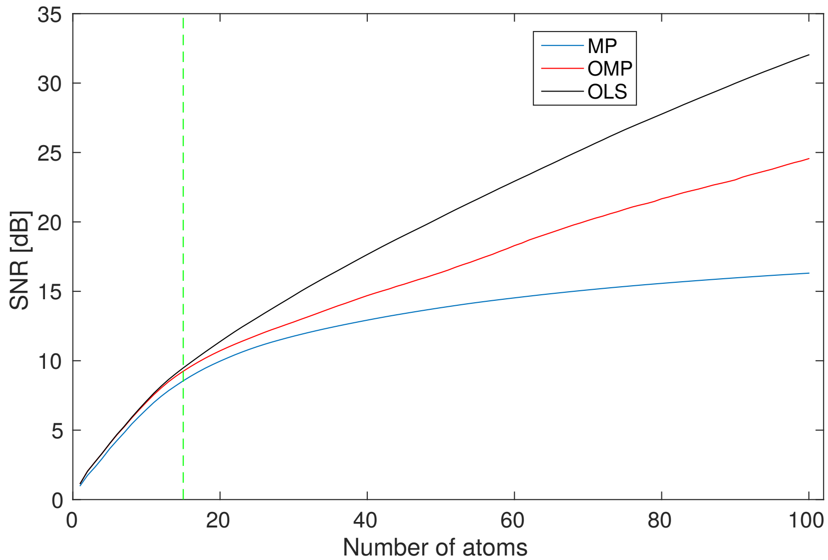
(a) Original shot gather.

(b) Reconstructed shot gather.

(c) Residues.

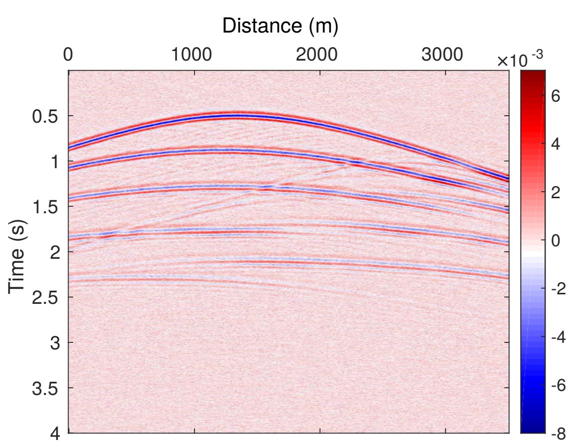
Multiple simulations were developed to determine the relationship between the SNR as the number of atoms increase (figure 10). It is possible to observe a similar behavior as in figure 6, where OLS obtained the best results in terms of $SNR/\#atoms$.

Figure 10: The SNR between the original and reconstructed shot gather by using the three MP algorithms for different amount of atoms.

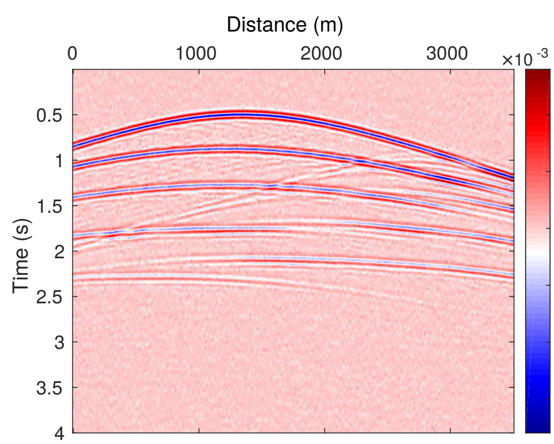


In a similar way, the noise analysis was developed on the shot gather from figure 9a. It was applied AWGN all over the shot with an SNR of 1 [dB] (figure 11a) and then, the OMP process was developed on it using 57 atoms (figure 11b). It was obtained an SNR of 10.4 [dB] between the original and reconstructed shot gather.

Figure 11: OMP process on a shot gather with 1 [dB] AWGN using 57 atoms.



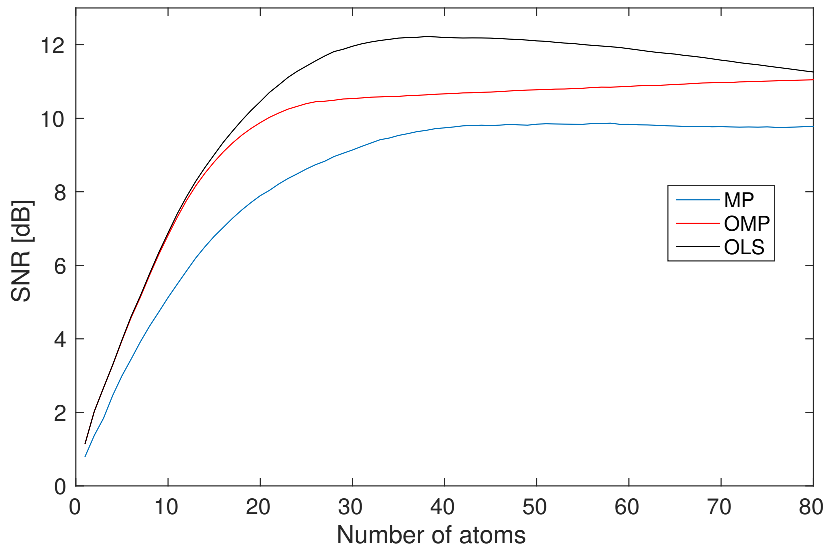
(a) Shot gather with AWGN.



(b) Reconstructed shot gather by OMP.

Figure 12 presents the responses obtained by applying the three MP algorithms on the previous shot gather with noise. The behaviors are quite similar to the one obtained with a seismic trace with AWGN (figure 8) but without presenting random jumps.

Figure 12: The SNR obtained by applying the three MP algorithms on the shot gather with noise for different amount of atoms.



Finally, 50 experiments were performed to estimate the mean value of SNR for different values of noise (table 2).

Table 2: SNR obtained after applying the three MP algorithms to a shot gather with different noise values.

Tested noise values	MP	OMP	OLS
3 [dB]	9.4 [dB]	12.4 [dB]	15.2 [dB]
1 [dB]	9.2 [dB]	11.2 [dB]	12.5 [dB]
-3 [dB]	8.1 [dB]	9.3 [dB]	10.6 [dB]

3. KIRCHHOFF MIGRATION OF COMPRESSED SEISMIC DATA

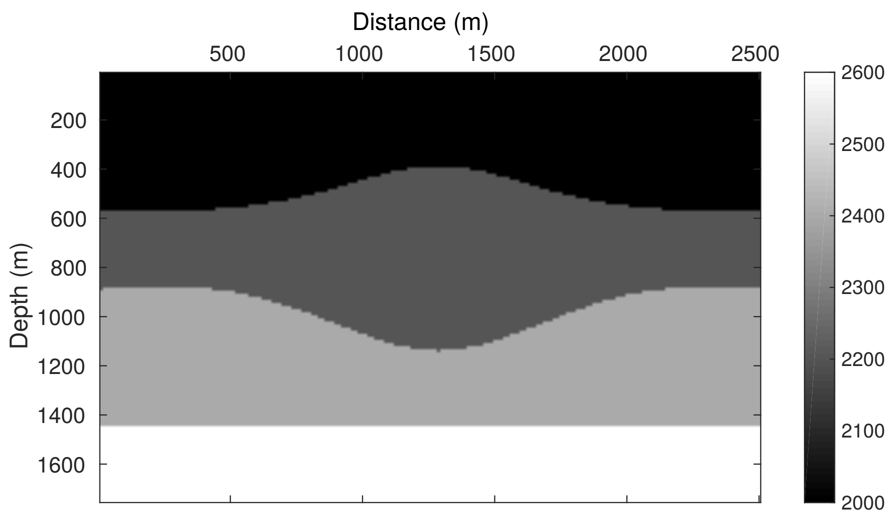
In this chapter, we expose the proposed strategies to develop the Kirchhoff migration on the seismic data compressed by the Matching Pursuit algorithms presented in the previous chapter. We denote the compression ratio (CR) as the relationship between the size of seismic data and the size of the compressed data, defined as:

$$CR = \frac{nt \cdot nx \cdot ns}{2 \cdot \#atoms \cdot nx \cdot ns} = \frac{nt}{2 \cdot \#atoms}, \tag{3.1}$$

where nt is the number of samples per seismic trace, nx is the number of receivers or seismic traces (without CPML) and ns is the number of shots.

Two strategies were proposed to develop the Kirchhoff migration on compressed seismic data. These strategies were tested taking into account the velocity model shown in figure 13.

Figure 13: Four layers velocity model.



The model is composed of four layers with constant density and a size of 2500 $\times$ 1875 [m] (200 $\times$ 140 points). The seismic data were modeled by a finite difference scheme of 2^{nd} order in time and space. The total amount of processed shot gathers was 11. The parameters used to model the data were Δx = Δz = 12.5 [m], Δt = 2 [ms] and 10 [Hz] for the source frequency.

3.1 Strategy I: Migrating the coefficients

The traditional Kirchhoff operator requires for some amplitudes of the seismic traces in time to map them into depth according to the travel time tables requirements (see Chapter 1). When the seismic data are compressed by some of the Matching Pursuit algorithms, we obtain the positions γ^k with their corresponding solution vector α^k (see Chapter 2). The γ^k coefficients represent the positions of the selected atoms from the dictionary and the Kirchhoff operator requires for positions in time. Thus, in order to perform the Kirchhoff migration, it is necessary to develop a domain transformation from Ricker domain to time domain.

This domain transformation can be achieved by analyzing how the dictionary is built up. As mentioned, the dictionary is created by taking an atom and applying a combination processes of scaling, frequency modulation and translations on it. In our case, we took an atom and applied a translation process sample by sample, i.e:

$$g_{\gamma}^{i+1} = g_{\gamma}^{i}(n-1) \tag{3.2}$$

Therefore, each position γ_i can be transformed into time by the following equation:

$$t^k = \gamma^k \cdot \Delta t + td, \tag{3.3}$$

where Δt is the sample rate of the seismic traces and td is the time where the maximum peak occurs in the first atom from the dictionary. On the other hand, it is also possible to build the dictionary by shifting the atoms by p samples, with $p > 1$, which would reduce the size of the dictionary. Based on this, the equation (3.3) can be rewritten as:

$$t^k = p \cdot \gamma^k \cdot \Delta t + td \tag{3.4}$$

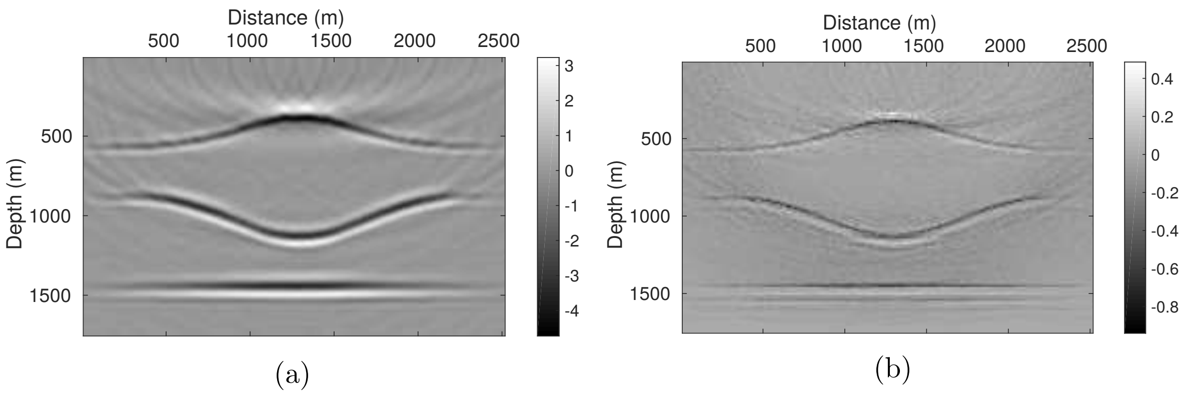
Thus, the Kirchhoff migration on compressed traces implies to modify the equation (1.2) as follows:

$$M(x,z) = \sum_{k=1}^{ns} \sum_{j=1}^{nr} w(r_j, s_k, x, z, tt) \sum_{i=0}^{m-1} C(r_j, s_k, \gamma_i, \alpha_i, td, tt), \tag{3.5}$$

where C represents the set of coefficients obtained from the compression stage, which are transformed and mapped according to the equation (3.3) and the travel time requirements respectively. Note that the transformation process is developed in the inner loop of the migration process.

Figure 14 shows the result of the proposed strategy using OMP as compression method with a CR of 70.

Figure 14: Migrated images obtained by a) the traditional Kirchhoff migration and b) the strategy I.



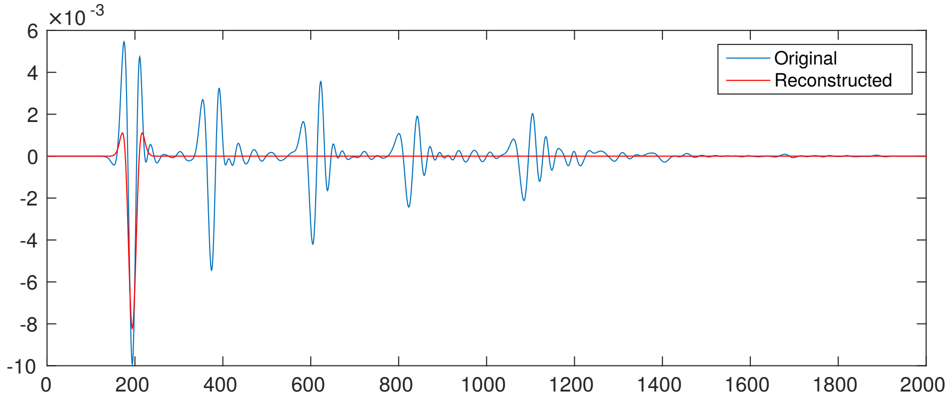
It is possible to observe that the migrated image obtained by the proposed strategy locates the reflectors in the same position as the traditional method but, there are two significant weaknesses: (i) the continuity of the reflector is strongly affected and, (ii) the amplitude values in the reflectors are reduced about 80 %. These seismic attributes are important for the geophysical expert at the moment of determining the structure and characteristics of the subsurface. However, this result was helpful because it allowed us to realize that the Ricker waveform associated to each γ_i and α_i had not been taking into account during the migration process, thus the strategy could be improved. In the next subsection we explain further this idea.

3.2 Strategy II: Migrating the atoms

Looking more closely in the migrated image obtained by the strategy I, it is possible to notice that the image is pixelated. This result can be explained because the migration process was developed into some parts of the seismic traces, i.e, the compression process performed a “sub-sampling” on the seismic data and then we migrated the resultant samples. The reason why the reflectors were well located, and more or less visible, was because the OMP algorithm selected the most relevant samples from the original seismic data. In this case, these samples contained a significant portion of the energy associated to the reflectors. Thus, the strategy I develops the traditional Kirchhoff migration but, on some samples from the original data taking α_k as the energy to map, which do not correspond to the correct amplitude values. A possible alternative to improve the strategy would be adding more relevant samples and figure out how to adjust the amplitude values but, we would be wasting the coded information contained in the coefficients (amplitude and phase of the Ricker waveform).

Based on this premise, the second strategy focuses on processing the information contained into the coefficients and not by mere samples. In figure 15 we present an example to explain this idea. The OMP process was performed on a seismic trace where

Figure 15: OMP process on a seismic trace using 1 atom.



the stopping criterion was 1 atom. As mentioned, the output would be a γ_0 and its corresponding α_0. However, these coefficients represent a Ricker waveform located at some point with an amplitude (the reconstructed signal). Thus, the strategy II performs mathematical operations on the coefficients during the migration process in order to map the red signal, instead of just mapping the coefficients.

The strategy II to perform the Kirchhoff migration can be divided as follows:

- **Transform from Ricker domain to travel time domain:** In the same way that strategy I, it is necessary to transform the γ^k into time domain because of the Kirchhoff operator. However, it is also possible to directly transform into travel time domain instead of time, i.e, just compute the samples required by the travel time tables and not all the reconstructed signal. This can be done by including γ^k, α^k and td into the Ricker equation eq. (2.7) as follows:

$$R_i(tt) = \alpha_i(1 - 2(tt - (\gamma_i + td))^2 f^2 \pi^2) e^{-(tt-(\gamma_i+td))^2 f^2 \pi^2}, \tag{3.6}$$

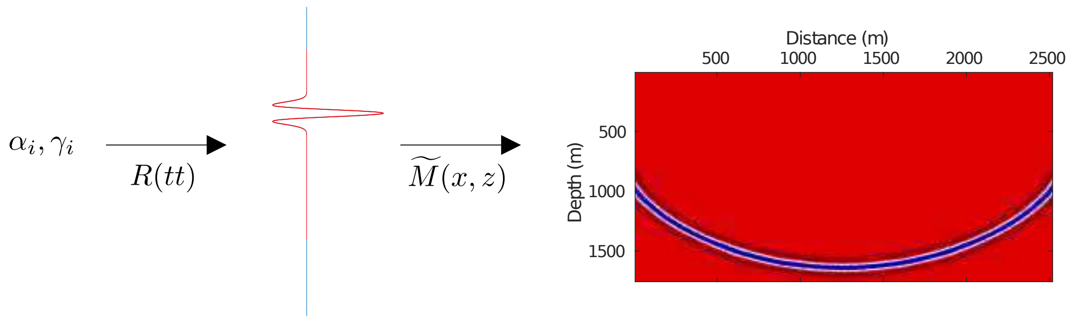
where tt are the samples required by the travel time tables.

- **Modified migration:** The traditional Kirchhoff method searches and maps samples from time to depth, which are the ones obtained by applying the eq. (3.6). During the mapping process, it is necessary to have in mind that the computed samples are part of a combination of Ricker waveforms that are associated with a respective source and receptor. According to this, the eq. (1.2) is modified as follows:

$$\widetilde{M}(x,z) = \sum_{k=1}^{ns} \sum_{j=1}^{nr} w(r_j, s_k, x, z, tt) \sum_{i=0}^{m-1} R(r_j, s_k, \alpha_i, \gamma_i, td, tt), \tag{3.7}$$

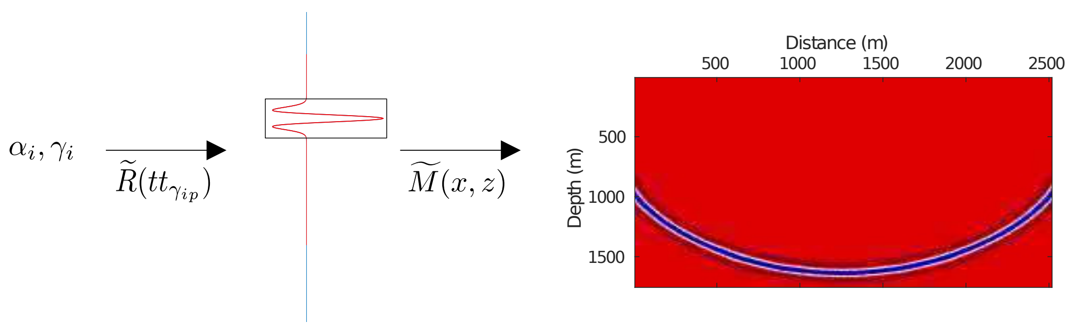
Figure 16 presents an example of the strategy II applied on two coefficients. The red line represents the samples of the signal required by the travel time tables which, in most of the cases, are less than the total amount of samples. However, most of the computed samples have a value of zero which did not contribute energy to the image.

Figure 16: Initial Strategy II.



Therefore, a more fitting strategy is to compute only the portion of the Ricker waveform that provide energy to the image, i.e, the samples located in the lobules of the Ricker (figure 17). With this, we reduce the amount of mathematical operations to be performed and save computational effort during the migration process.

Figure 17: Final Strategy II.



Thus, the equation (3.7) can be rewritten as follows:

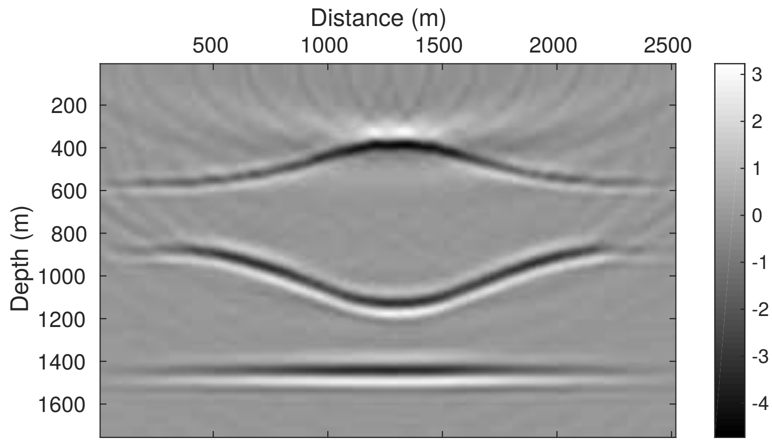
$$\widetilde{M}(x,z) = \sum_{k=1}^{ns} \sum_{j=1}^{nr} w(r_j, s_k, x, z, tt_{\gamma_{ip}}) \sum_{i=0}^{m-1} \widetilde{R}(r_j, s_k, \alpha_i, \gamma_i, td, tt_{\gamma_{ip}}), \tag{3.8}$$

where $tt_{\gamma_{ip}}$ represent the samples required by the travel time tables within the period of each atom.

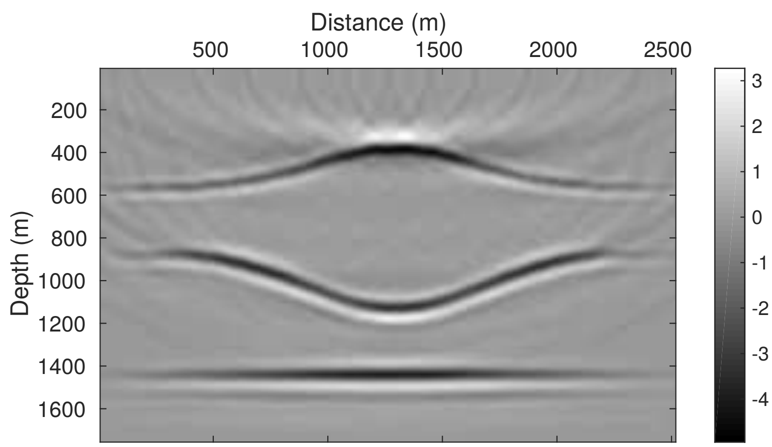
Note that, although the use of this strategy implies to reconstruct some parts of the atoms, the samples of $\widetilde{R}(tt_{\gamma_{ip}})$ are directly computed in the image. By doing this, we achieve one of the main objectives of this work, that was to eliminate the reconstruction (decompress) stage before performing the seismic migration.

Figure 9 presents the result of the proposed strategy using OMP with a CR of 70. Note that, no discontinuity is observed in the reflectors and the amplitude values are

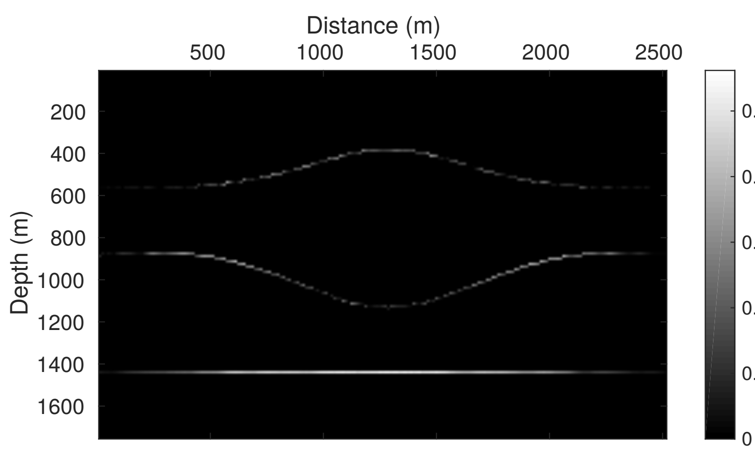
Figure 18: Migrated images obtained by a) the traditional Kirchhoff migration, b) the strategy II and c) the residuals.



(a)



(b)



(c)

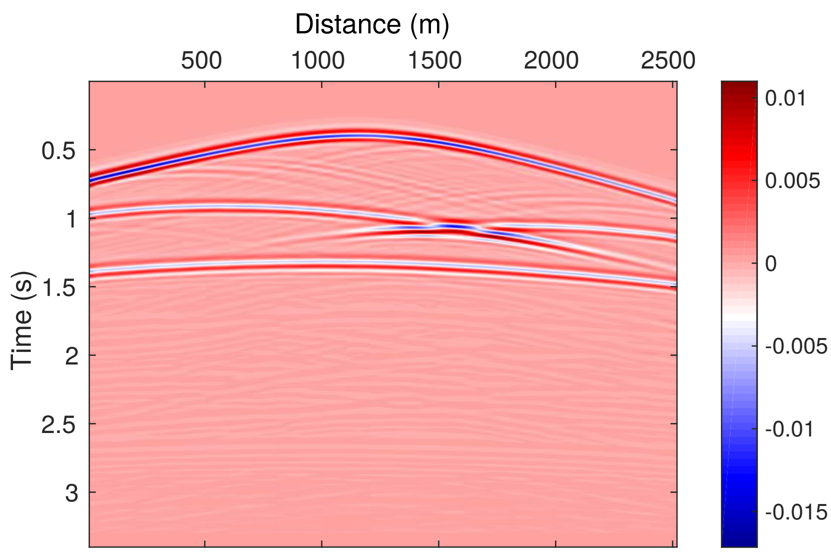
not significantly affected. The maximum error was $0.54 \approx 15\%$, located in the deepest reflector (figure 18c).

From this result, the principal aim of the migration process has been successfully achieved, i.e, with a CR of 70 (12 atoms per trace) it was possible to correctly locate the reflectors and keep the continuity and amplitude values in comparison to the standard method.

Moreover, from this result, it was estimated a possible relationship between the number of atoms employed to compress the seismic data and the structure of the obtained image. More specifically, it was possible to approximately determine the minimum number of atoms necessary to obtain, at least, the same structure produced by the standard method. This can be done by taking into account the number of seismic events presented in the shot gathers and the velocity model, before performing the compression process.

Figure 19 presents one of the shot gathers obtained from the modeled data. It is possible to observe in detail four seismic events in the shot gather with some dispersion. This response is due to the three velocity changes in the velocity model and the "tie effect" that produce the synclinal reflector. Then, these four seismic events provided most of the energy associated with the reflectors obtained in figure 18a, at the moment of performing the migration process.

Figure 19: Middle shot gather from the velocity model of 4 layers.



According to the shape of the Ricker waveform, each seismic event is represented by 3 lobules and, in practice, each lobule can be approximately represented by one atom from the dictionary. This is the reason why 12 atoms were used to compress the seismic data in the previous experiment. Therefore, we propose the following equation to estimate the possible maximum CR that ensure to conserve the structure of the migrated image:

$$CR_s \approx 3 \cdot k \cdot ncv, \tag{3.9}$$

where CR_s represents the maximum compression ratio, k is a constant factor between 1–2 that depends on the complexity of the velocity model and the response obtained in the shot gathers and, ncv is the number of visibles strong changes of velocity. Our results show that, in most of the cases, with a $k = 2$ it was guaranteed the location and continuity of the reflectors. In the next chapter, we test this equation with two additional velocity models and also, we use other metrics to measure the quality of the results for different values of CR.

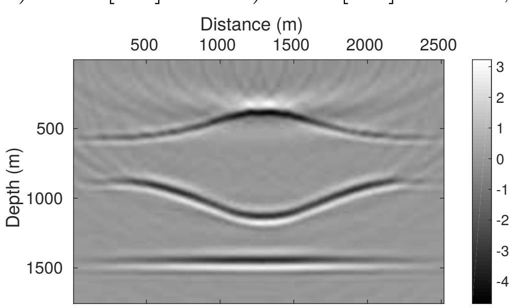
4. RESULTS

In this chapter three velocity models are proposed to test the quality of the results obtained by the Kirchhoff migration using the strategy II. The metrics employed to measure the quality were the Signal-Noise Ratio (SNR), the percentage amplitude error in the reflectors and the frequency spectrum between the migrated images obtained by the proposed and traditional method. The compression factor and the three Matching Pursuit algorithms were taken as independent variables to analyze their implications on the mentioned metrics.

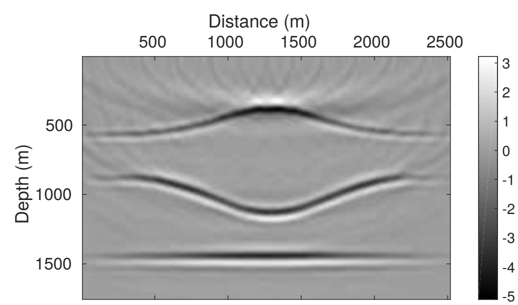
4.1 Model of 4 layers

The velocity model is the same used in the previous chapter (figure 13). In figure 20, the migrated image obtained by the standard Kirchhoff migration is compared in terms of SNR with the ones obtained by the proposed migration.

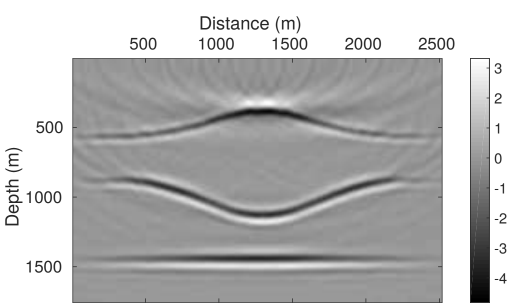
Figure 20: Migrated images obtained by the a) stardard and b-d) proposed Kirchhoff migration using the model of four layers. The SNR between the images are b) 7.8 [dB], c) 11.4 [dB] and d) 11.2 [dB] for MP, OMP and OLS respectively.



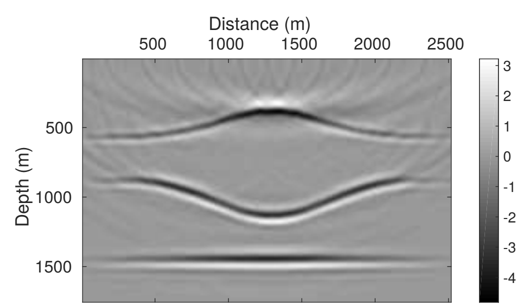
(a) Traditional Kirchhoff migration.



(b) Kirchhoff migration using MP.



(c) Kirchhoff migration using OMP.

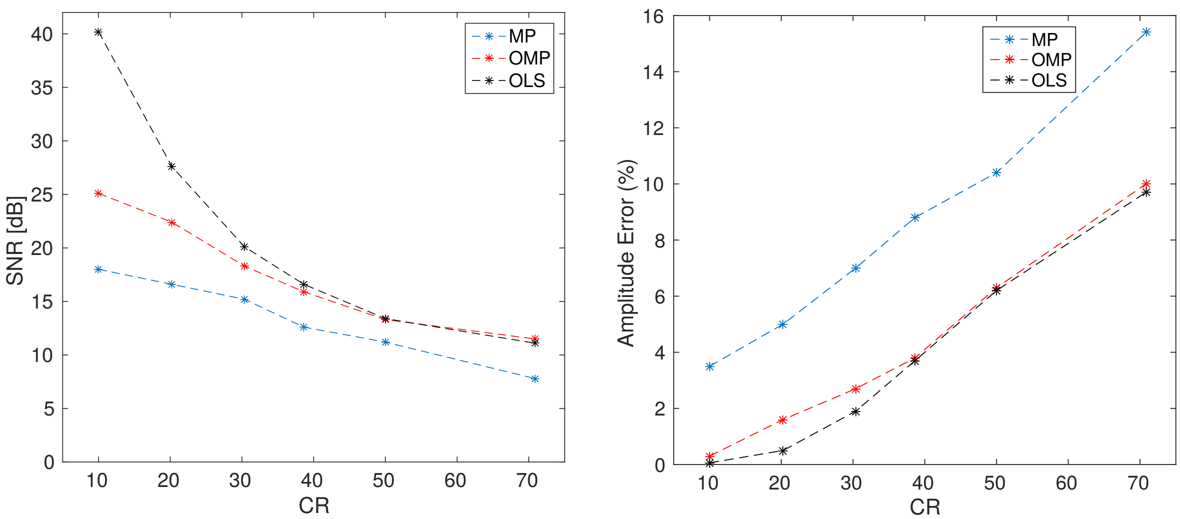


(d) Kirchhoff migration using OLS.

In this experiment, the seismic data were compressed based on the equation (3.9), yielding a CR of 70. It is possible to observe that the location, continuity and amplitude of the reflectors are conserved using the three Matching Pursuit algorithms. In terms of SNR, OMP obtained the best result.

Multiple experiments were developed to determine the relationship between the SNR and CR (figure 21a). The SNR obtained by OLS presents a decremental exponential behavior, while OMP and MP were approximately linear. Above of a compression ratio of 50, OLS gave the best results in terms of SNR in comparison to the other methods, with a maximum of 40.2 [dB] for a CR of 10.

Figure 21: Relationship between a) SNR vs CR and b) the percentual error in the amplitude of reflectors vs CR obtained from the model of 4 layers.



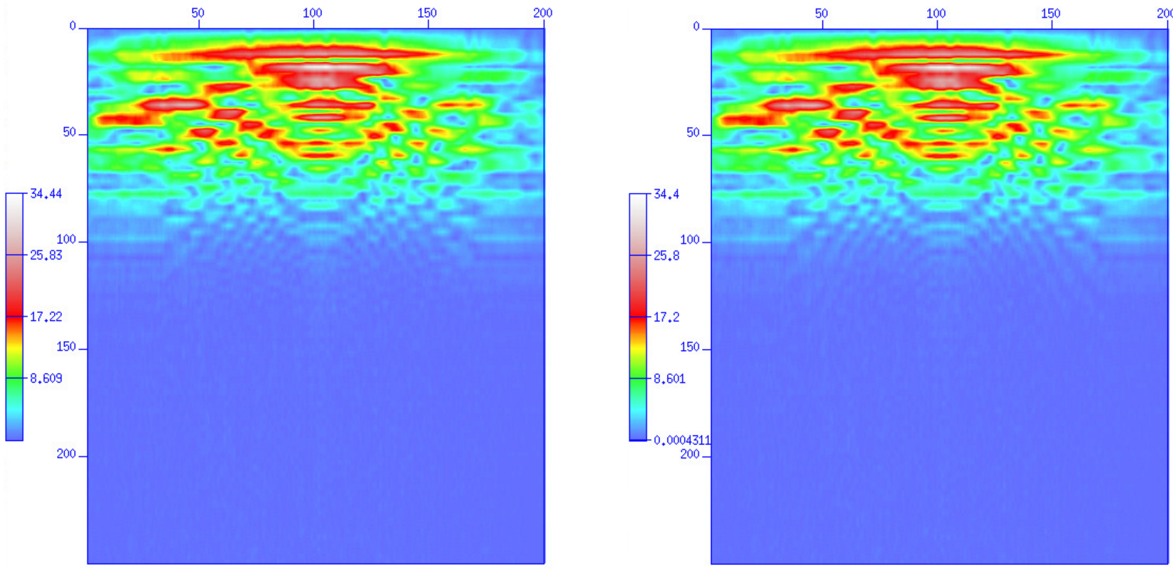
(a) (b)

Additionally, in figure 21b is presented the percentual changes in the amplitude of the reflectors vs CR. The norm$-l2$ between the migrated images was taken as metric to measure the error. OLS got the best results in comparison with the others, with a minimum error about 0.07 % for a CR of 10. The three MP algorithms approximately presented a linear behavior.

Finally, it was performed a frequency analysis between the migrated images obtained by the traditional Kirchhoff migration and the proposed method, using the SU command *suspecfx*. In figure 22a, is presented the Fourier spectrum of figure 20a meanwhile, in figure 22b is presented the Fourier spectrum of the migrated image obtained with OMP and a CR of 20. It is possible to observe that the shape is relatively conserved as well as the magnitude, with an error under 0.1 %.

Table 3 presents the time spent to compress the seismic data by the three Matching Pursuit algorithms. Note that, the time spent to compress the seismic data by OLS is significantly higher than the other methods for any CR. As mentioned before, OLS

Figure 22: Frequency spectrum of the migrated images obtained by the a) standard and b) proposed migration using OMP with a CR of 20.



(a) (b)

requires a high computational cost because of the multiple amounts of matrix operations. Approximately, OMP and MP spent 0.1 % and 0.04 % of the time spent by OLS to develop the compression stage respectively.

Table 3: Time spent to compress a) 1 shot gather and b) 11 shot gathers.

CR	MP	OMP	OLS
70	0.2 s	0.3 s	4 min
50	0.3 s	0.5 s	6 min
40	0.4 s	0.8 s	9 min
30	0.5 s	1 s	13 min
20	1 s	2 s	28 min
10	2.3 s	8 s	1.7 h

(a)

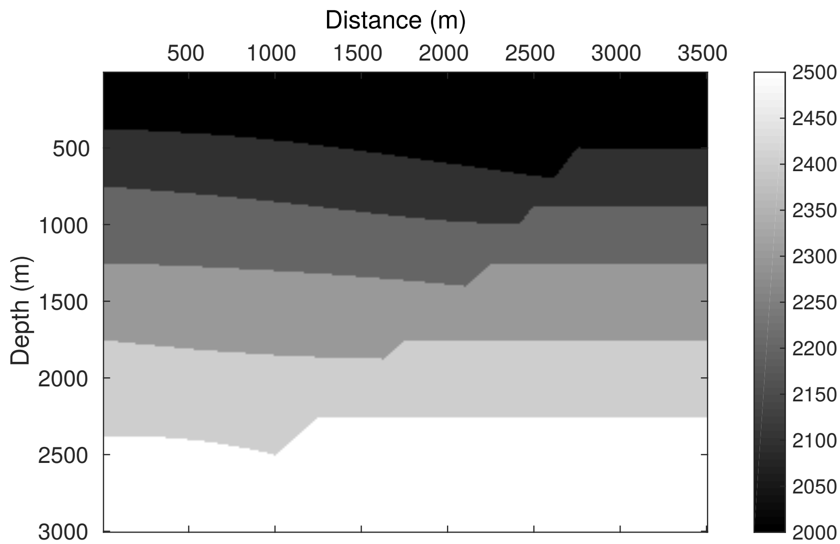
CR	MP	OMP	OLS
70	2.5 s	3.6 s	44 min
50	3.2 s	6 s	1.1 h
40	4 s	9 s	1.7 h
30	6 s	12 s	2.5 h
20	11 s	23 s	5 h
10	26 s	85 s	19 h

(b)

4.2 Model of 6 layers

This second velocity model was designed to have more reflectors (subsurface layers) with a common real-life geometry. This model is bigger than the previous one, with a size of 3500×3000 [m] (280×240 points), two more reflector and double seismic fault, each one with a different inclination angle (figure 23). In this case, 6 shot gathers were processed. The seismic data was modeled using the same strategy and parameters than the previous model.

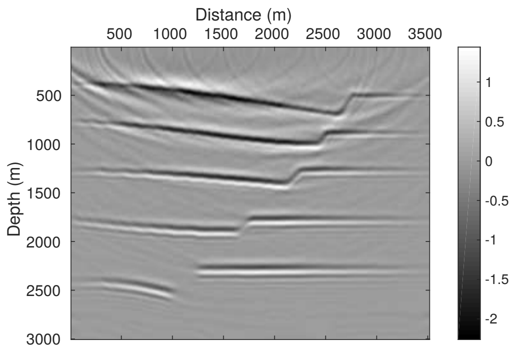
Figure 23: Velocity model of six layers.



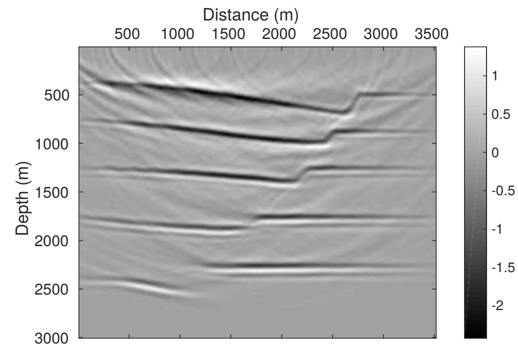
In this experiment, the compression ratio was selected based on the equation (3.9). Taking a value of $k = 1$, the seismic data was compressed with a CR about 76. In figure 24, the migrated image obtained by the standard Kirchhoff migration is compared in terms of SNR with the ones obtained by the proposed migration using the three Matching Pursuit algorithms. The location, continuity and amplitude of the reflectors are relatively conserved. In this case, the best result in terms of SNR was also obtained by using OLS as compression method.

In a similar way, multiple experiments were developed to determine the relationship between the SNR and the CR (figure 25a). In this case, OLS and OMP presented a decremental exponential behavior while MP was approximately linear. The results show that OLS obtained the best results in terms of SNR in comparison to the others methods, with a maximum of 41 [dB] for a compression ratio of 10. Additionally, in figure 25b is presented the percentual changes in the amplitude of the reflectors vs CR. Again, OLS got the best results in comparison with the other methods, with a minimum error about 0.08 %. The three MP algorithms approximately presented a linear behavior.

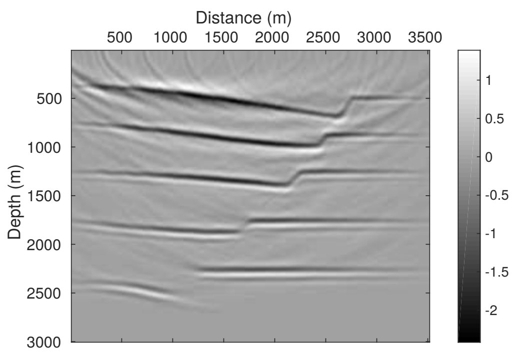
Figure 24: Migrated images obtained by a) stardard and b-d) proposed Kirchhoff migration using the model of six layers. The SNR between the images are b) 7.7 [dB], c) 9.1 [dB] and d)10.8 [dB] for MP, OMP and OLS respectively..



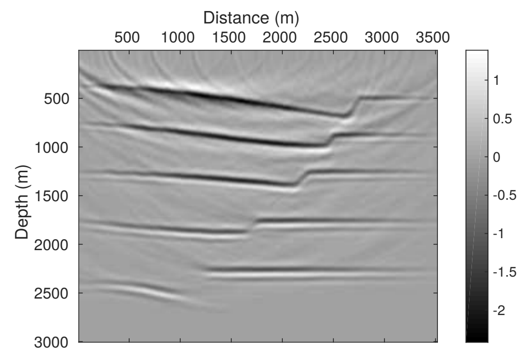
(a) Traditional Kirchhoff migration.



(b) Kirchhoff migration using MP.

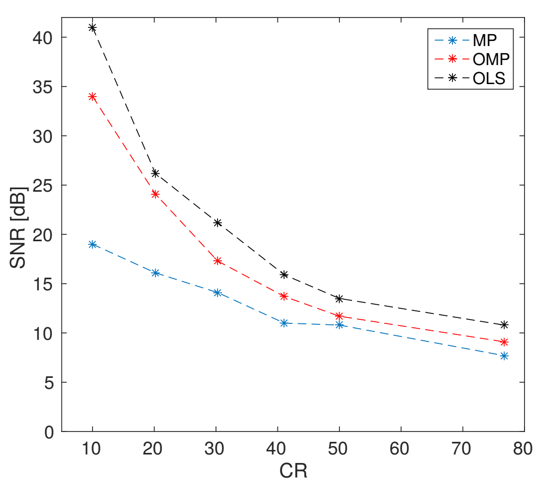


(c) Kirchhoff migration using OMP.

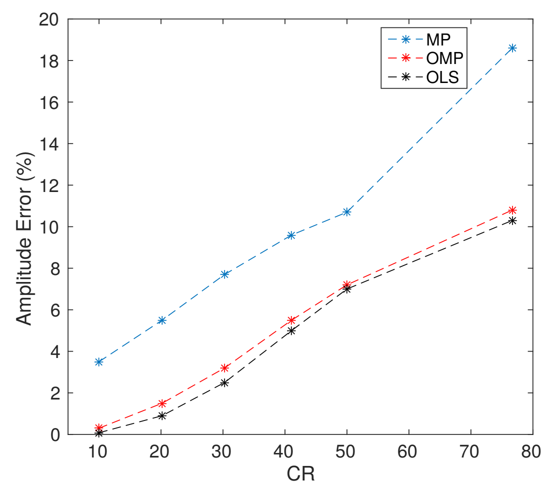


(d) Kirchhoff migration using OLS.

Figure 25: Relationship between a) SNR vs CR and b) the percentual error in the amplitude of reflectors vs CR obtained from the model of 6 layers.



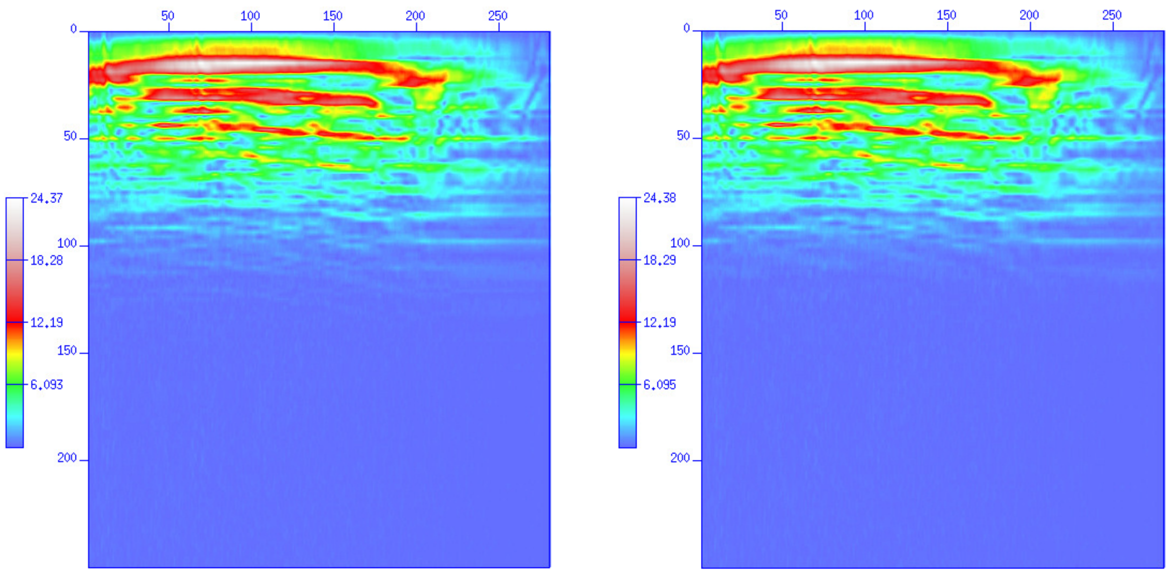
(a)



(b)

The frequency analysis between the migrated images obtained by the traditional Kirchhoff migration and the proposed method is presented in figure 26. In figure 26a is presented the Fourier spectrum of figure 24a meanwhile, in figure 26b is presented the Fourier spectrum from the migrated image obtained with OMP and 20 of CR. It is possible to observe that the shape is relatively conserved as well as the magnitude, with an error under 0.1 %.

Figure 26: Frequency spectrum of the migrated images obtained by the a) standard and b) proposed migration using OMP with a CR of 20.



(a) (b)

Table 4 presents the times spent by each one of the Matching Pursuit algorithms to compress the seismic data. Approximately OMP and MP spent 0.1 % and 0.05 % of the time spent by OLS to develop the compression stage respectively.

Table 4: Time spent to compress a) 1 shot gather and b) 9 shot gather.

CR	MP	OMP	OLS
76	0.7 s	1 s	12 min
50	1 s	1.5 s	23 min
40	1.3 s	2 s	40 min
30	1.7 s	3 s	55 min
20	2.7 s	7 s	2.2 h
10	7.3 s	28 s	13 h

(a)

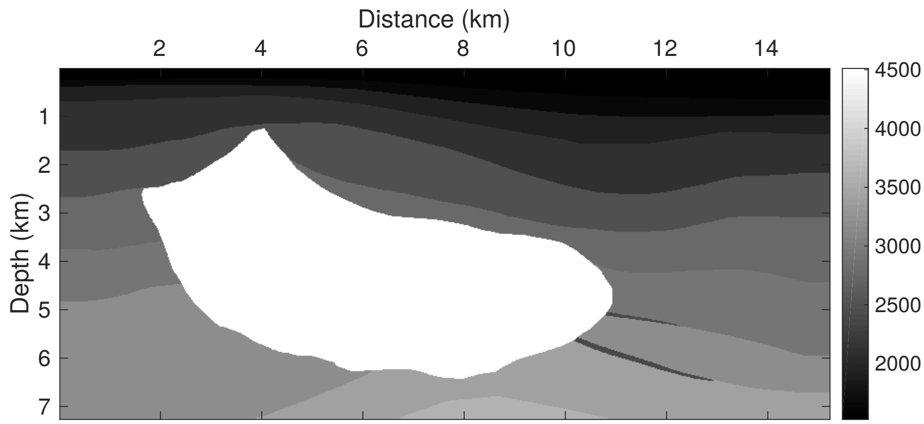
CR	MP	OMP	OLS
76	5 s	10 s	1.8 h
50	9 s	14 s	3.4 h
40	11 s	18 s	6 h
30	15 s	30 s	8.2 h
20	24 s	60 s	20 h
10	66 s	250 s	5 d

(b)

4.3 SEG/Hess model

Finally, a portion of the 2D SEG/Hess velocity model was used to test the proposed method. This is a real model which presents a high complex structure to work with. This model has a size of 7.2 × 14.4 [km] (721× 363 points), eight layers and a saline dome with two wedges (figure 27). It was taken a total amount of 81 shot gathers to process. It was taken a Δx = Δz = 20 [m], Δt = 2 [ms] and 10 [Hz] for the frequency of the source. It should be mentioned that the elastic parameters of the model were not taken into account to obtain the seismic data.

Figure 27: SEG/Hess velocity model.

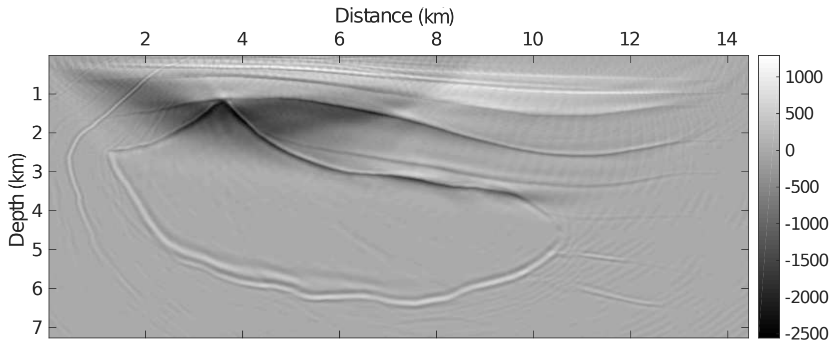


In this section, we test the proposed Kirchhoff migration by only using OMP as compression method. The reason is that the Hess model is approximately three times bigger than the two velocity models tested and, based on the results obtained in those experiments, performing the OLS process would take approximately a month, while, the OMP process showed acceptable results in terms of quality/execution time.

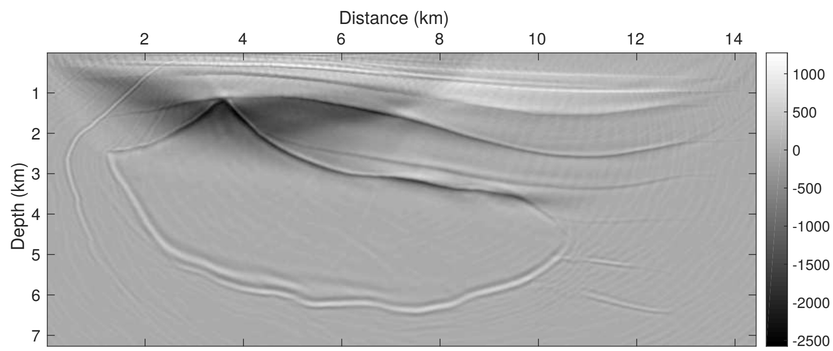
Figure 28a presents the migrated images obtained by the standard Kirchhoff migration and figure 28b shows the image obtained by the proposed migration using OMP. The compression ratio was also selected based on the equation (3.9), yielding a CR of 58 for a $k = 2$. It is possible to observe that the continuity, amount and position of the reflectors were conserved in comparisson with the traditional. The amplitude values are relatively conserved with a 3 % of error and, the SNR between the migrated images was about 14 [dB]. It should be mentioned that, OMP compressed each shot gather in 1 minute.

The frequency analysis between the migrated images are presented in figure 29. Figure 29a presents the Fourier spectrum of the image from figure 28a meanwhile, figure 29b shows the Fourier spectrum from the migrated image obtained in figure 28b. It is possible to observe that the shape is relatively conserved as well as the amplitude values, with an error of 5 %.

Figure 28: Hess sections obtained by the a) standard and b) proposed Kirchhoff migration using OMP.

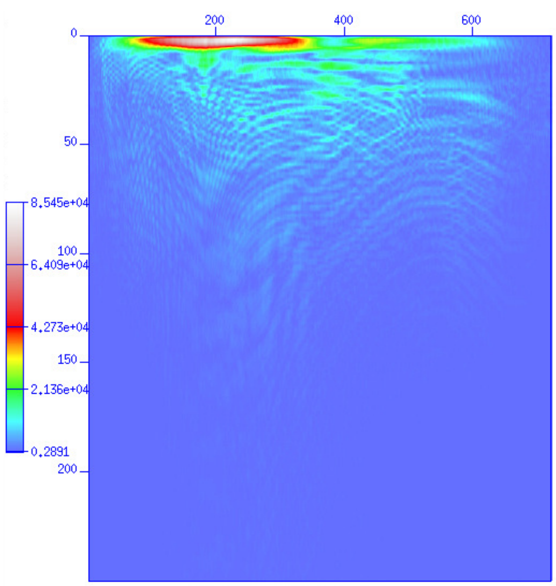


(a)

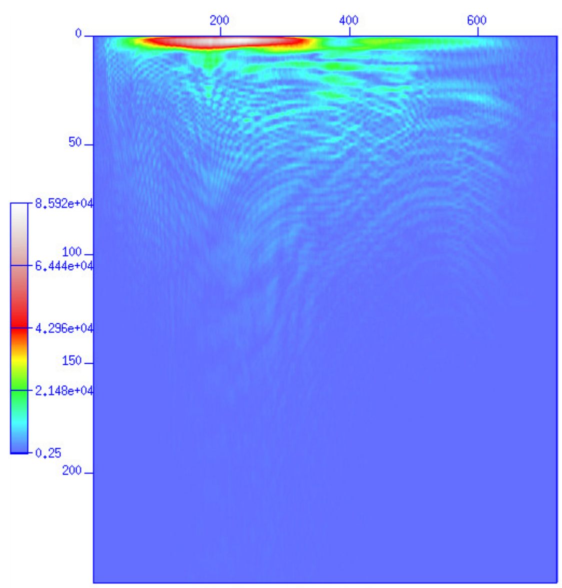


(b)

Figure 29: Frequency spectrum of the migrated images obtained by the a) standard and b) proposed migration using OMP with a CR of 58.



(a)



(b)

5. CONCLUSIONS

A 2D pre-stack Kirchhoff migration has been performed on compressed seismic data by using three different Matching Pursuit algorithms. The fact that the Matching Pursuit algorithms compress the seismic data into few coefficients (γ^k and α^k), which contain coded Ricker waveforms, allowed us to modify the Kirchhoff operator to perform this process in a compressed domain. No significant changes in quality were observed in the migrated images obtained by our method in comparison to the traditional. The tests were developed using different velocity models, as well as, synthetic seismic data which were modeled by a finite difference scheme. The traditional and the proposed Kirchhoff migration were performed in Matlab, while the seismic data were obtained by using a GPU.

Currently, seismic data compression algorithms are required in order to reduce the transmission and storage costs. However, the decompression process in the computer centers increases the processing time. The proposed strategy in this work eliminates the reconstruction stage before the Kirchhoff migration by directly mapping the computed samples into the image.

In the compression stage, it was analyzed each Matching Pursuit algorithm used in terms of SNR vs # of atoms. In this case, as the computational complexity of the compression algorithms increased, it was possible to represent better the seismic data. Our results showed that the OLS algorithm offers the best results in terms of SNR for any amount of atoms in comparison to MP and OMP. On the other hand, it was shown that in presence of AWGN in the seismic data, the compression methods filtered most of the noise, increasing the SNR up to 13 [dB] using OLS.

The quality of the results was determined by taking into account the seismic attributes of the migrated images obtained by the second proposed strategy. In qualitative terms, the continuity and location of the reflectors of the migrated images were conserved, in comparison with the ones obtained by the traditional method. Additionally, it was proposed an equation to estimate the minimum amount of atoms necessary to compress seismic data that ensure to maintain the continuity and location of the reflectors. Moreover, the frequency analysis showed no significant changes in shape for both low and high frequency.

In quantitative terms, the magnitude error between the Fourier spectrums of the migrated images was less than 0.1% for a compression ratio of 20. Furthermore, the results in terms of SNR and percentual error in the amplitude of the reflectors suggest employing OLS as compression method. Using OLS, it was possible to obtain an SNR above of 40 [dB] with an error in the amplitude less than 0.1 % for a compression ratio of 10. However, it is important to mention that OLS spent a considerable amount of time to perform the compression process in comparison with the other two methods (MP and OMP). More specifically, the relationship in terms of time between OLS and the others was about in the order of 1000:1. This is because of the thousand of matrix operations that have to be developed during the selection process of OLS.

On the other hand, OMP obtained acceptable results in terms of quality and it is much faster than OLS. The results in terms of processing time were in the order of minutes, not hours (or even days) as OLS and, the values of SNR and amplitude error in the reflectors showed to conserve enough the seismic attributes of the migrated images. The MP method offered a faster compression method than OMP and OLS, in the order of seconds, but the quality of the results was not good enough.

Finally, the proposed method demonstrate an acceptable performance on real velocity models, such as the SEG/Hess model which, presents challenges to work with because of its complex structures.

Future Work:

It is clear that nowadays it is more expensive, in terms of execution time, to develop memory operations than mathematic operations at the moment of processing a big amount of data. The proposed strategy aims to search for replacing memory operations by math operations (such as sums, subtractions, multiplications, etc..), which can significantly reduce the processing times for big data applications. It is expected to implement this strategy in a heterogeneous cluster (based on GPUs) to observe the impacts of this strategy on the overall processing time. We will look for performing an implementation with an amount of data that overcome the node memory because of the reading/writing operations in the disk are the most computationally expensive.

Moreover, we propose to implement the OLS process in a GPU in order to develop the selection process (matrix operations) in a parallel manner, which can decrease the execution time of the algorithm. Finally, we encourage the idea of extrapolating the Kirchhoff migration on compressed data to a 3D scenario and also, to consider the idea of developing another migration process in a compressed domain, such as the Reverse Time Migration (RTM).

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APPENDIX

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