Hoja de diseño teórico para la geometría de un motor tipo bates con perfil de quemado neutral

Propellant independent parameters --

burning rate

$r = 11,3 \cdot \frac{1}{1000}$ *burn time m/s*

Specific Impulse

$ISP = 140 \cdot 0,93$

Impulso especifico en segundos corregido, correccion tomada de bibliografia

Densidad

$\rho_p = 1750$ *kg / m3*

ratio of specific heats

$k = 1,042$

Mission/vehicle independent parameters --

$D_{grain} = 82 \cdot \left| 0,001 \cdot \frac{\cdot m}{mm \cdot} \right|$ ---

$go = 9,81$ [m/s^2]

total rocket mass, kg

$mt = 22$ [kg]

Burn time

$tp = 2,4$

$P_c = 1100 \cdot \left| 6895 \cdot \frac{\cdot Pa}{psi \cdot} \right|$

$P_e = 95000$ [Pa]

$P_{amb} = P_e$

Diseño del grano--

Launch wieght:

$wt = mt \cdot go$ *peso total del cohete*

Useful propellant weight:

$w = m \cdot go$ *peso del propelente eyectado*

mass ratio

$mo = wt$

$mf = wt - w$

$mR = \frac{mf}{mo}$ *relacion entre el peso final e inicial del cohete*

specific impulse

$ISP = \frac{F}{\dot{w}}$

$It = F \cdot tp$

$F_{lbs} = F \cdot 0{,}224809$

$It_{lbs} = F_{lbs} \cdot tp$

web fraction

$W_f = \frac{2 \cdot r \cdot 2{,}43}{D_{grain}}$

$W_f = \frac{D_{grain} - dd}{D_{grain}}$ *con esta ecuación se calcula el diámetro interno ideal*

$Lg = 1 / 2 \cdot [3 \cdot D_{grain} + dd]$

longitud ideal del grano para obtener un perfil de quemado neutro, con un área de quemado inicial y final igual

cálculo de la longitud total en base a la masa necesaria

$\rho_p = \frac{m}{vol}$

$vol = 1 / 4 \cdot \pi \cdot [D_{grain}^2 - dd^2] \cdot Lfin$

$N = \frac{Lfin}{Lg}$

este es el número ideal de segmentos a utilizar, se acerca a un valor entero por criterio del diseñador

Estudio balistico---

Launch angle

$\theta = 85$

$\dot{w} = \frac{w}{tp}$

The thrust is obtained:

$Fo = ISP \cdot \frac{w}{tp}$

The initial accelarations along the x and y are:

$$a_{oy} = go \cdot \left[Fo \cdot \frac{\sin(\theta)}{wt} - 1 \right]$$

$$a_{ox} = go \cdot Fo \cdot \frac{\cos[\theta]}{wt}$$

At thrust termination the initial flight acceleration becomes:

$$a_o = \sqrt{a_{oy}^2 + a_{ox}^2}$$

Velocities

$$c = go \cdot ISP$$

$$Upy = c \cdot \ln\left[\frac{mo}{mf}\right] \cdot \sin[\theta] - go \cdot tp$$

$$Upx = c \cdot \ln\left[\frac{mo}{mf}\right] \cdot \cos[\theta]$$

$$Uo = \sqrt{Upy^2 + Upx^2}$$

$$Uokmh = Uo \cdot 3600 \cdot \frac{1}{1000}$$

For the powered flight, the coordinates at propulsion burnout Yp and Xp can be calculated from the time integration of their integration of their respective velocities:

$$Ypp = 1 / 2 \cdot go \cdot tp^2$$

$$Yp = go \cdot ISP \cdot \left[1 - \left(\frac{\ln\left[\frac{mo}{mf}\right]}{\frac{mo}{mf} - 1} \right) \right] \cdot \sin[\theta] - Ypp$$

$$Xp = go \cdot ISP \cdot \left[1 - \left(\frac{\ln\left[\frac{mo}{mf}\right]}{\frac{mo}{mf} - 1} \right) \right] \cdot \cos[\theta]$$

The umpowered part of the trajectory reaches zero vertical velocity at its zenith. The light gained in unpowered free flight may be obtained by equating the vertical kinetic energy at power cutoff to its equivalent potential energy

$CCF = 1,5$ *correction due to friction avoidance*

$$Yz = 3000 \cdot CCF$$

3 km, 1,5 factor de correccion

$$go \cdot [Yz - Yp] = 1 / 2 \cdot Upy^2$$

The maximun height or zenith location thus becomes:
What remins now is to solve the free-flight

portion of vertical descent.The time for ascent from the zenith

$$Yz = 1 / 2 \cdot go \cdot tz^2$$

The impact vertical velocity

$$Ufy - go \cdot tz = 0$$

During free-flight the horizontal velocity remains unchanged at 70,6 ft/s because there are no accelarations (no drag, wind, or gravity component)

The time from burnout to the zenit:

$$Upy = go \cdot t$$

The total free-flight time becomes:

$$tff = t + tz$$

Now, the total range becomes:

$$\delta X = Xp + Upx \cdot tff$$

Diseño de la tobera--

Ideal Thrust coefficient Cf

$$C_F = \sqrt{\left[\left(\frac{2 \cdot k^2}{k - 1} \right) \cdot \left(\left[\frac{2}{k + 1} \right]^{\left[\frac{k + 1}{k - 1} \right]} \right) \right] \cdot \left[1 - \left(\frac{P_e}{P_c} \right)^{\left(\frac{k - 1}{k} \right)} \right]} + \left[\frac{P_e - P_{amb}}{P_c} \right] \cdot \varepsilon$$

$$\varepsilon = \frac{A_e}{A_t}$$

$$A_t = \pi \cdot \left[\frac{dt}{2} \right]^2$$

epsilon = 10

$$A_e = \pi \cdot \left[\frac{de}{2} \right]^2$$

$$de = 50 \cdot \left| 0{,}001 \cdot \frac{\cdot m}{mm \cdot} \right|$$

P_e= exit plane pressure ; epsilon= nozzle area expansion ratio A_e/A_t

$$C_{Fvac} = \sqrt{\left[\left(\frac{2 \cdot k^2}{k - 1} \right) \cdot \left(\left[\frac{2}{k + 1} \right]^{\left[\frac{k + 1}{k - 1} \right]} \right) \right] \cdot \left[1 - \left(\frac{P_e}{P_c} \right)^{\left(\frac{k - 1}{k} \right)} \right]} + \left[\frac{P_e - 0}{P_c} \right] \cdot \varepsilon$$

$$C_{Fact} = \lambda \cdot \eta_f \cdot \left[C_{Fvac} - \frac{P_e}{P_c} \cdot \varepsilon \right] + \left[\frac{P_e - P_{amb}}{P_c} \right] \cdot \varepsilon$$

lambda is the nozzle divergence correction factor *for conical nozzle*

$\lambda = 1 / 2 \cdot [1 + \cos(\alpha)]$

$\alpha = 15$ *nozzle divergence half angle, deg*

eta is the C_f efficiency factor

$\eta_f = 0{,}9$

$Fo = P_c \cdot A_t \cdot C_{Fact}$

SOLUTION

Unit Settings: [J]/[C]/[Pa]/[kg]/[degrees]

dd = 0,02708	de = 0,05	dt = 0,01674	D_{grain} = 0,082
F = 2579	ISP = 130,2 [0]	It = 6189	Lfin = 0,5885
Lg = 0,1365	m = 4,846 [0]	mR = 0,7797 [0]	mt = 22 [kg]
N = 4,31	P_c = 7,584E+06 [Pa 0]	t = 29,87	tp = 2,4 [0]
Uo = 294,3 [0]	Uokmh = 1060 [0]	Yz = 4500 [0]	

37 potential unit problems were detected.