APPENDIX 2

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APPENDIX 2

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Abstract

This appendix presents the comprehensive mathematical framework supporting the design of the RC oscillator. Due to the significant length and algebraic complexity of the exact analytical expressions, they were omitted from the main body of the manuscript to preserve the flow and readability of the text.

The following sections detail the full derivation of the oscillation frequency equation, incorporating non-ideal parameters such as switch on-resistances (R_{on}), comparator open-loop gain limitations, offset voltages, and Schmitt trigger propagation delays (t_d). Furthermore, this section compiles the complete set of partial derivatives used for the sensitivity analysis. These expressions, derived and verified using symbolic computation tools (MATLAB), serve as the theoretical basis for the validation results presented in the Modeling section.

1 Sensitivity analysis for each non-ideal parameter in the frecuency equation

RESISTOR

$$\frac{\partial F}{\partial R}=\frac{C\,\log\left[\frac{V_{\mathrm{off}}-V_{\mathrm{TH}}+\frac{V_{\mathrm{DD}}}{A_0}}{V_{\mathrm{DD}}\left(\exp\left(-\frac{t_{d2}}{C\,R_{\mathrm{ONP}}}\right)-1\right)+\exp\left(-\frac{t_{d2}}{C\,R_{\mathrm{ONP}}}\right)\left(V_{\mathrm{off}}-V_{\mathrm{TH}}+\frac{V_{\mathrm{DD}}}{A_0}\right)}\right]}{2\,t_{d2}-C\,\log\left[\frac{V_{\mathrm{off}}-V_{\mathrm{TH}}+\frac{V_{\mathrm{DD}}}{A_0}}{V_{\mathrm{DD}}\left(\exp\left(-\frac{t_{d2}}{C\,R_{\mathrm{ONP}}}\right)-1\right)+\exp\left(-\frac{t_{d2}}{C\,R_{\mathrm{ONP}}}\right)\left(V_{\mathrm{off}}-V_{\mathrm{TH}}+\frac{V_{\mathrm{DD}}}{A_0}\right)}\right](R+R_{\mathrm{ONN}})}$$

RONN

$$\frac{\partial F}{\partial RONN}=\frac{C\,\log\left[\frac{V_{\mathrm{off}}-V_{\mathrm{TH}}+\frac{V_{\mathrm{DD}}}{A_0}}{V_{\mathrm{DD}}\left(\exp\left(-\frac{t_{d2}}{C\,R_{\mathrm{ONP}}}\right)-1\right)+\exp\left(-\frac{t_{d2}}{C\,R_{\mathrm{ONP}}}\right)\left(V_{\mathrm{off}}-V_{\mathrm{TH}}+\frac{V_{\mathrm{DD}}}{A_0}\right)}\right]}{2\,t_{d2}-C\,\log\left[\frac{V_{\mathrm{off}}-V_{\mathrm{TH}}+\frac{V_{\mathrm{DD}}}{A_0}}{V_{\mathrm{DD}}\left(\exp\left(-\frac{t_{d2}}{C\,R_{\mathrm{ONP}}}\right)-1\right)+\exp\left(-\frac{t_{d2}}{C\,R_{\mathrm{ONP}}}\right)\left(V_{\mathrm{off}}-V_{\mathrm{TH}}+\frac{V_{\mathrm{DD}}}{A_0}\right)}\right](R+R_{\mathrm{ONN}})}$$

RONP

$$\frac{\partial F}{\partial RONP}=\frac{\frac{C(R+R_{\mathrm{ONN}})\left[V_{\mathrm{DD}}\cdot t_{d2}\cdot\exp\left(-\frac{t_{d2}}{CR_{\mathrm{ONP}}}\right)+t_{d2}\cdot\exp\left(-\frac{t_{d2}}{CR_{\mathrm{ONP}}}\right)\left(V_{\mathrm{off}}-V_{\mathrm{TH}}+\frac{V_{\mathrm{DD}}}{A_0}\right)\right]}{(CR_{\mathrm{ONP}})^2}}{\left[2\,t_{d2}-C\,\log\left(\frac{V_{\mathrm{off}}-V_{\mathrm{TH}}+\frac{V_{\mathrm{DD}}}{A_0}}{V_{\mathrm{DD}}\left(\exp\left(-\frac{t_{d2}}{C\cdot R_{\mathrm{ONP}}}\right)-1\right)+\exp\left(-\frac{t_{d2}}{C\cdot R_{\mathrm{ONP}}}\right)\left(V_{\mathrm{off}}-V_{\mathrm{TH}}+\frac{V_{\mathrm{DD}}}{A_0}\right)}\right)(R+R_{\mathrm{ONN}})\right]^2\cdot\left[V_{\mathrm{DD}}\left(\exp\left(-\frac{t_{d2}}{CR_{\mathrm{ONP}}}\right)-1\right)+\exp\left(-\frac{t_{d2}}{CR_{\mathrm{ONP}}}\right)\left(V_{\mathrm{off}}-V_{\mathrm{TH}}+\frac{V_{\mathrm{DD}}}{A_0}\right)\right]}$$

CAPACITOR

$$\frac{\partial F}{\partial C}=\frac{\left[\log\left(\frac{V_{\mathrm{off}}-V_{TH}+\frac{V_{DD}}{A0}}{V_{DD}\left(e^{\frac{td2}{C\cdot R_{\mathrm{ONP}}}}-1\right)+e^{\frac{td2}{C\cdot R_{\mathrm{ONP}}}}\left(V_{\mathrm{off}}-V_{TH}+\frac{V_{DD}}{A0}\right)}\right)\right]\cdot(R+R_{\mathrm{ONN}})-\frac{C\cdot(R+R_{\mathrm{ONN}})\cdot\left[\frac{V_{DD}\cdot td2\cdot e^{\frac{td2}{C\cdot R_{\mathrm{ONP}}}}}{C^2\cdot R_{\mathrm{ONP}}}+\frac{td2\cdot e^{\frac{td2}{C\cdot R_{\mathrm{ONP}}}}\left(V_{\mathrm{off}}-V_{TH}+\frac{V_{DD}}{A0}\right)}{C^2\cdot R_{\mathrm{ONP}}}\right]}{V_{DD}\left(e^{\frac{td2}{C\cdot R_{\mathrm{ONP}}}}-1\right)+e^{\frac{td2}{C\cdot R_{\mathrm{ONP}}}}\left(V_{\mathrm{off}}-V_{TH}+\frac{V_{DD}}{A0}\right)}}{\left[2\,td2-C\cdot\log\left(\frac{V_{\mathrm{off}}-V_{TH}+\frac{V_{DD}}{A0}}{V_{DD}\left(e^{\frac{td2}{C\cdot R_{\mathrm{ONP}}}}-1\right)+e^{\frac{td2}{C\cdot R_{\mathrm{ONP}}}}\left(V_{\mathrm{off}}-V_{TH}+\frac{V_{DD}}{A0}\right)}\right)\cdot(R+R_{\mathrm{ONN}})\right]^2}$$

VTH

$$\frac{\partial F}{\partial V_{TH}}=\frac{A_0^2\cdot C\cdot V_{DD}\cdot(R+R_{\mathrm{ONN}})\cdot\left[e^{\frac{td2}{C\cdot R_{\mathrm{ONP}}}}-1\right]}{\left[2\,td2-C\,(R+R_{\mathrm{ONN}})\cdot\log\left(\frac{V_{DD}-A_0\,V_{TH}+A_0\,V_{\mathrm{off}}}{V_{DD}+A_0\,V_{DD}-A_0\,V_{TH}+A_0\,V_{\mathrm{off}}-A_0\,V_{DD}\cdot e^{\frac{td2}{C\cdot R_{\mathrm{ONP}}}}}\right)+\frac{td2}{C\cdot R_{\mathrm{ONP}}}\right]^2\cdot(V_{DD}-A_0\,V_{TH}+A_0\,V_{\mathrm{off}})\cdot\left[V_{DD}+A_0\,V_{DD}-A_0\,V_{TH}+A_0\,V_{\mathrm{off}}-A_0\,V_{DD}\cdot e^{\frac{td2}{C\cdot R_{\mathrm{ONP}}}}\right]}$$

Voff

$$\frac{\partial F}{\partial V_{off}}=\frac{A_0^2\cdot C\cdot V_{DD}\cdot(R+R_{\mathrm{ONN}})\cdot\left[e^{\frac{td2}{C\cdot R_{\mathrm{ONP}}}}-1\right]}{\left[2\,td2-C\,(R+R_{\mathrm{ONN}})\cdot\log\left(\frac{V_{DD}-A_0\,V_{TH}+A_0\,V_{\mathrm{off}}}{V_{DD}+A_0\,V_{DD}-A_0\,V_{TH}+A_0\,V_{\mathrm{off}}-A_0\,V_{DD}\cdot e^{\frac{td2}{C\cdot R_{\mathrm{ONP}}}}}\right)+\frac{td2}{C\cdot R_{\mathrm{ONP}}}\right]^2\cdot(V_{DD}-A_0\,V_{TH}+A_0\,V_{\mathrm{off}})\cdot\left[V_{DD}+A_0\,V_{DD}-A_0\,V_{TH}+A_0\,V_{\mathrm{off}}-A_0\,V_{DD}\cdot e^{\frac{td2}{C\cdot R_{\mathrm{ONP}}}}\right]}$$

VDD

$$\frac{\partial F}{\partial VDD}=-\frac{A_0^2\cdot C\cdot(R+R_{\mathrm{ONN}})\cdot(V_{TH}-V_{\mathrm{off}})\cdot\left[e^{\frac{td2}{C\cdot R_{\mathrm{ONP}}}}-1\right]}{\left[2\,td2-C\,(R+R_{\mathrm{ONN}})\cdot\log\left(\frac{V_{DD}-A_0\,V_{TH}+A_0\,V_{\mathrm{off}}}{V_{DD}+A_0\,V_{DD}-A_0\,V_{TH}+A_0\,V_{\mathrm{off}}-A_0\,V_{DD}\cdot e^{\frac{td2}{C\cdot R_{\mathrm{ONP}}}}}\right)+\frac{td2}{C\cdot R_{\mathrm{ONP}}}\right]^2\cdot(V_{DD}-A_0\,V_{TH}+A_0\,V_{\mathrm{off}})\cdot\left[V_{DD}+A_0\,V_{DD}-A_0\,V_{TH}+A_0\,V_{\mathrm{off}}-A_0\,V_{DD}\cdot e^{\frac{td2}{C\cdot R_{\mathrm{ONP}}}}\right]}$$

A0

$$\frac{\partial F}{\partial A_0}=\frac{C\cdot V_{DD}^2\cdot(R+R_{\mathrm{ONN}})\cdot\left[e^{\frac{td2}{C\cdot R_{\mathrm{ONP}}}}-1\right]}{\left[2\,td2-C\,(R+R_{\mathrm{ONN}})\cdot\log\left(\frac{V_{DD}-A0\,V_{TH}+A0\,V_{\mathrm{off}}}{V_{DD}+A0\,V_{DD}-A0\,V_{TH}+A0\,V_{\mathrm{off}}-A0\,V_{DD}\cdot e^{\frac{td2}{C\cdot R_{\mathrm{ONP}}}}}\right)+\frac{td2}{C\cdot R_{\mathrm{ONP}}}\right]^2\cdot(V_{DD}-A0\,V_{TH}+A0\,V_{\mathrm{off}})\cdot\left[V_{DD}+A0\,V_{DD}-A0\,V_{TH}+A0\,V_{\mathrm{off}}-A0\,V_{DD}\cdot e^{\frac{td2}{C\cdot R_{\mathrm{ONP}}}}\right]}$$

td2

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$$\frac{\partial F}{\partial td2} = \frac{\dfrac{C\,(R + R_{\mathrm{ONN}})\left[\dfrac{e^{-\frac{td2}{C\,\cdot\,R_{\mathrm{ONP}}}}\left(V_{\mathrm{off}} - V_{TH} + \frac{V_{DD}}{A0}\right)}{C\,\cdot\,R_{\mathrm{ONP}}} + \dfrac{V_{DD}\,e^{-\frac{td2}{C\,\cdot\,R_{\mathrm{ONP}}}}}{C\,\cdot\,R_{\mathrm{ONP}}}\right]}{V_{DD}\left(e^{-\frac{td2}{C\,\cdot\,R_{\mathrm{ONP}}}} - 1\right) + e^{-\frac{td2}{C\,\cdot\,R_{\mathrm{ONP}}}}\left(V_{\mathrm{off}} - V_{TH} + \frac{V_{DD}}{A0}\right)} - 2}{\left[2\,td2 - C\,\log\left(\dfrac{V_{\mathrm{off}} - V_{TH} + \frac{V_{DD}}{A0}}{V_{DD}\left(e^{-\frac{td2}{C\,\cdot\,R_{\mathrm{ONP}}}} - 1\right) + e^{-\frac{td2}{C\,\cdot\,R_{\mathrm{ONP}}}}\left(V_{\mathrm{off}} - V_{TH} + \frac{V_{DD}}{A0}\right)}\right)(R + R_{\mathrm{ONN}})\right]^2}$$