ANÁLISIS DEL PROBLEMA DE DIMENSIONAMIENTO DE FLOTA DE VEHÍCULOS SUJETO A UNA DEMANDA ESTOCÁSTICA

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Trabajo de Grado para optar por el título de
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RESUMEN

TITULO: ANÁLISIS DEL PROBLEMA DE DIMENSIONAMIENTO DE FLOTA DE VEHÍCULOS SUJETO A UNA DEMANDA ESTOCÁSTICA[*]

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PALABRAS CLAVES: DIMENSIONAMIENTO DE FLOTA, DEMANDA ESTACIONAL, PLNE, ESTUDIO DE CASO

Proyecto de grado modalidad Investigación realizado en colaboración con el Departamento de Ingeniería Industrial de la Universidad TOBB Economics & Technology situada en Ankara, Turquía.

El dimensionamiento y la configuración de la flota son dos problemas de decisión fundamentales para todas las empresas de distribución en el mundo actual. El problema del dimensionamiento estratégico de la flota es muy complejo debido a sus numerosas limitaciones en el transporte, almacenamiento y cálculo de costos totales.

El objetivo de este proyecto de investigación es proporcionar una metodología comprensible para el problema de dimensionamiento del tamaño de la flota de vehículos con diferentes restricciones como la demanda estacional, la configuración de la flota, las redes de transporte y las ventanas de tiempo en el servicio.

En una primera parte de este estudio, se presenta un estado del arte para el problema de dimensionamiento del tamaño de la flota en el que se introducen varios modelos de optimización con sus respectivos enfoques de solución. Seguido de ello, se introduce un estudio de caso real de un distribuidor de muebles y accesorios para el hogar que desea consolidar sus actividades de almacenamiento y distribución en un solo almacén.

Se propone un modelo de programación lineal entera mixta para determinar el número total de vehículos propios y alquilados, y su capacidad para ser asignada a cada región a fin de satisfacer la demanda de los clientes en un intervalo de tiempo dado. Se estudian diferentes escenarios con el fin de determinar el impacto de la variación de la demanda, la capacidad de los vehículos, el coste de propiedad y de alquiler de vehículos, y la duración del intervalo de tiempo permitido para las entregas sobre el tamaño final de la flota. Los resultados del modelo determinista propuesto dan información valiosa para la toma de decisiones estratégicas.

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SUMMARY

TITLE: ANALYSIS OF THE FLEET SIZING PROBLEM UNDER STOCHASTIC AND SEASONAL DEMAND[*]

AUTHOR: SILVIA CATALINA GONZÁLEZ FLÓREZ[**]

KEYWORDS: FLEET SIZING, SEASONAL DEMAND, MILP, CASE STUDY

Graduation project in Research Operations conducted within the department of Industrial Engineering at TOBB University of Economics & Technology located in Ankara, Turkey.

Fleet sizing and fleet configuration are fundamental decision problems for all distribution companies in today's world. The problem of the strategic fleet sizing is extremely challenging because of its numerous constraints in transportation, storage and total costing.

The objective of this study is to provide a comprehensible methodology for the fleet sizing problem under different constraints such as seasonal demand, fleet configuration, transport networks and service time windows.

First chapter of this study addresses a general state of art for the fleet sizing problem where several optimization models are introduced with their respective solution approaches. The second chapter is based on a real case study of a main furniture and home accessories distributor who wants to consolidate the storage and shipment activities into one warehouse.

A mixed integer linear program is proposed to determine the total number of owned and rented vehicles and their capacity to be assigned to each region in order to satisfy the customers demand in a given time interval. Different scenarios are studied to see the impact of the demand variation, the vehicles' capacity, the owning and rental cost of vehicles and the length of the time interval allowed for the deliveries on the fleet sizes. The results of the proposed deterministic model give valuable information for strategic decision-making.

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INTRODUCTION

During the last decades, the freight transportation has become one of the most important issues for distribution businesses because of its significant impact in the total cost of end products, as well as in the quality of service. According to [Ghiani *et al.* 2004], it can be considered as strategic level decision because of its long-lasting effects and the acquisition of costly resources, like the number of vehicles in a fleet. It also requires medium to long term data collection from several parties involved in the logistic chain such as producers, distributors, retailers, truck rental firms and customers. Freight transportation can be referred as the integration of quality and service where distributors are expected to supply customers by providing the right product, during the appropriate period of time and at the agreed place.

It also allows to find the best compromise between the fleet size and the service level. Determining the optimal fleet size and its composition is directly related to finding the trade-off between the sum of investment, the operating cost of the fleet and the cost of not satisfying the customers demand in required time intervals. Freight transportation companies should focus on managing their fleets in order to achieve a complete fulfillment of the incoming transportation orders and avoid high fixed costs associated with fleet underutilization and unsatisfied or backlogged demands.

In order to cover demand, two alternatives can be implemented according to each particular case; owning a complete fleet of vehicles or renting one part of it from an outsider firm – rental firms. The first case is associated to a major investment cost and it is commonly used when covering deterministic demand. On the contrary, the second case is mostly preferred when covering stochastic and seasonal demand, and it has a less significant investment cost. Renting option could be very advantageous specially in case of covering peak demand periods. In this study, these two types of fleet configuration are considered and the number of owned and rented vehicles is computed according to seasonal demand structure for a major furniture and home accessories distributor. The aim of the distributor is to centralize the storage and distribution activities into one warehouse in order to have a standardized service level for its customers. Having no initial fleets to ensure the demand dispatching, the distributor seeks to find the optimal fleet sizing to acquire and/or to rent during low and high season. The main contributions of this study are the introduction of a comprehensible methodology for finding the number and the type of owned and rented fleet of vehicles from a real-life distribution system and to show the impact of demand variation, vehicles' capacity, service time window length and cost parameters on the final decisions of the fleet configuration.

This report is structured in the following way. Chapter 1 provides a literature survey summarizing the current state of the art related to the fleet sizing problem. Chapter 2 describes the real-life problem of the furniture and home accessories distributor. Chapter 3 presents the fleet sizing problem considered in here with the respective assumptions and data analysis. The mathematical model for the problem is detailed in Chapter 4, in the form of a single objective mathematical programming model. To evaluate the performance of the proposed model, in Chapter 5, different scenarios are studied regarding several factors of the furniture and home accessories distributor with extensive discussion of the results. Finally, Chapter 6 summarizes the primary contributions, provides conclusions and recommendations for future researches.

1. General State-Of-The-Art

Reviewing the recent literature, several models for vehicle fleet management have been proposed and then they have been classified according to different criteria. One of the most important issues in fleet management is the Fleet Sizing Problem, which focuses on satisfying the customer demand by determining the optimal vehicle fleet size. Having a large size of fleet suggests higher fixed costs, while not having enough vehicles to supply customers implicates an additional cost for unmet demand. A commonly used criteria to classify fleet sizing problems concerns the type of model, whether it is deterministic or stochastic.

According to [Ohnari 1998], a *deterministic model* has no stochastic elements and the relation between inputs and outputs of the model is conclusively determined. In other words, deterministic models consider that no randomness is involved and all parameters are known or assumed, e.g. parameters such as transport demand, supplying and traveling times are exact values. A *stochastic model* has one or more stochastic elements, which means the system is generally not solved analytically but using probability distributions. This type of model assumes obtaining different outputs for each input due to its random component.

For both approaches, deterministic and stochastic, the literature review showed that the fleet sizing problem is often addressed either integrated with the Vehicle Routing Problem or with the Fleet Assignment/Repositioning Problems, or independently. The classification of the studies found in the literature is given in Table 1.

According to [Singer *et al.* 2002], in the literature the problem of configuring a fleet with the determination of the number of vehicles and their capacities has been studied into two classes: analytical models and routing methods. Analytical models do not involve any routing and use approximations for route lengths while the routing methods use exact routing to solve the fleet sizing problem.

Table 1. Literature classification for the Fleet Sizing Problem

Criteria / Type of Model	Deterministic	Stochastic
Integrated with the Vehicle Routing Problem	Desrochers and Verhoog (1991) Salhi and Sari (1997)	Yang *et al.* (2000) Christiansen and Lysgaard (2007) Secomandi and Margot (2009)
Integrated with the Fleet Assignment and Repositioning Problem	Sayarshad and Ghoseiri (2008)	Singer *et al.* (2002) Köchel *et al.* (2003) Song and Earl (2008) Dong and Song (2009)
Independent from the Vehicle Routing Problem	Imai and Rivera (2001) Sayarshad *et al.* (2010) Redmer *et al.* (2012) Laake and Zhang (2013)	Turnquist and Jordan (1986) Novaes and Graciolli (1999) Papier and Thonemann (2008) Dong and Song (2012)

Several studies exist in the literature with deterministic demand/travel time assumptions for the fleet sizing problem, independent from the vehicle routing and repositioning problems. [Sayarshad and Ghoseiri 2008] propose a new solution method to optimize the fleet sizing and the freight car allocation. In this article, car demands and travel times are deterministic values while unmet demands are backordered. A Simulated Annealing algorithm is proposed to solve the model. [Imai and Rivera 2001] propose an analytical model to find the container fleet size between two ports with implicit empty container repositioning. They extend their study to many-to-many ports and construct a simulation model to determine the composition of owned and leased container fleet. Some other studies for the fleet sizing problem in the literature consider the railway industry. For example [Sayarshad *et al.* 2010] present a multi-objective mathematical model and a solution method for optimizing fleet strategic decisions. The three objectives are; minimization of the cost of unmet demands (service quality), maximization of the profit and minimization of the rail-car fleet size. The Pareto optimal set is used for a trade-off analysis due to the existence of conflicting objectives. Related with maritime transportation, [Laake and Zhang 2013] propose a mixed integer programming optimization model for strategic fleet planning in tramp shipping industry. Besides focusing on determining the best mix of contracts for a given fleet, the study also finds the optimal fleet size and its configuration. Another example from fuel distribution business is proposed by [Redmer *et al.* 2012]. In their study, they propose a model and a heuristic approach to the fleet composition problem for a 2-echelon distribution system. The purpose of the model is to compose an optimal fleet of tankers in a central depot of fuel distribution system. The authors make important assumptions such as, maximum capacity of vehicles, maximum working time of drivers, and the decision variables are assumed to be integer only. Three different heuristic procedures are used to solve the decision problem. They are based on local search, evolutionary algorithms and hybrid algorithms.

The fleet sizing problem has been treated with stochastic assumptions in the literature. For the stochastic fleet sizing problem independent of the vehicle routing and the repositioning assumptions, some studies are described here. [Turnquist and Jordan 1986] assume stochastic container travel times for a one manufacturer-multiple assembly plants system. They provide a model which determines the fleet size with a given probability of running short of containers and show the impact of uncertainty and the number of plants on the fleet size. [Novaes and Graciolli 1999] partition the region under study into sectors, ring and districts and compute the best vehicle size. Some relevant assumptions are maximum allowed travel time per day and vehicles' capacities, and vehicle cycle time and vehicle loads are treated probabilistically. Many other articles consider the Vehicle Routing Problem with stochastic demand, such as [Yang *et al.* 2000] and [Secomandi and Margot 2009]. In both cases, the classical approach of the Vehicle Routing Problem with stochastic demand is used to describe a simple return into the depot for replenishment of empty vehicles when defined routes are completed or a failure has occurred. [Christiansen and Lysgaard 2007] introduce an exact method for the vehicle routing problem with stochastic demands. They describe a new branch-and-price-based algorithm in which each customer has several copies. The vehicle capacity and the customer demand are simultaneously considered when creating a new copy of each node for each amount of product. Independently from the vehicle routing problem, [Papier and

Thonemann 2008] propose a model to determine the optimal fleet size and to optimize the fleet size structure for a leading European cargo rail company. Analytical models for rental fleet optimization are studied in the literature as well, in which uncertain demand, rental time, seasonality, and order batching are simultaneously considered. The paper of [Dong and Song 2012] is one of such studies where the authors propose a simulation-based optimization approach for the container fleet sizing under uncertain customer demand and stochastic inland transport times.

In the literature, the stochastic fleet sizing integrated with the repositioning problem of the empty vehicles integrated attracted many researchers. [Singer *et al.* 2002] study the problem of configuring a fleet on a Chilean company that distributes liquefied petroleum gas. They assume average demands, average distances and average visiting times to attend a customer, and they take into account working days per month for a given fleet. They use queuing systems to model the waiting times and propose a method to decide the number of vehicles and their capacity under different constraints. [Song and Earl 2008] focus on solving the problem of determining optimal control policies for empty vehicle repositioning and fleet-sizing in a two-depot service system with uncertainties in loaded vehicle arrival and repositioning times for empty vehicles. The model follows two stages: first, it assumes the fleet size as fixed in order to find the optimal empty repositioning policy; second, it determines the optimal fleet size and the total cost function. [Dong and Song 2009] consider the joint container fleet sizing and empty container repositioning problem in liner shipping systems with stochastic demand. The optimization solving method is based on Genetic Algorithms and Evolutionary Strategy. In the context of general fleet sizing and empty vehicle allocation, an extensively literature exists but only a few articles consider both problems simultaneously (see [Kochel *et al.* 2003]).

To finalize the state-of-the-art for this problem it is worth to give some examples for the integrated fleet sizing and vehicle routing problems which also represent important research topics in the literature. For instance, [Desrochers and

Verhoog 1991] explore the fleet size and mixed vehicle routing problem for a road transportation system with a central depot. The decision problem is defined as a graph where the optimal fleet composition and routes are defined for a heterogeneous fleet. The problem is solved by a two-phase procedure: in the first phase a traveling salesman algorithm is applied to assign arcs to each vehicle, and in the second phase, a heuristic route construction algorithm is applied. [Salhi and Sari 1997] study the vehicle routing and the fleet composition problems simultaneously for a multi-depot and multi-customer system under deterministic assumptions and propose an efficient heuristic.

Based on the literature review it is worth mentioning that no previous studies have been done on the determination of the deterministic fleet sizes assigned to predefined regions while taking into account the average annual demands at each retailer and different time constraints. Although there are some models with similar assumptions to the problem studied in here, no single model in the literature covers all the features of this particular problem. The optimization model proposed in this study is novel to describe a complex distribution problem for a real-life case and gives useful information for strategic decision-making to other companies wishing centralize their distribution activities.

2. Description of the case study

In this chapter, a case study of a major furniture and home accessories distributor is introduced as an example of fleet sizing problem. The aim is to represent the relevant costs such as the fixed costs (renting/owning costs) and the operational costs (routing costs); as well as all the constraints obtained from a real-life distribution system. Therefore, we analyze the impact of demand variation, vehicles’ capacity, costs parameters and time of service interval allowed for the shipments on the fleet size. In the following sections we describe the problem, some important assumptions, and the data analysis.

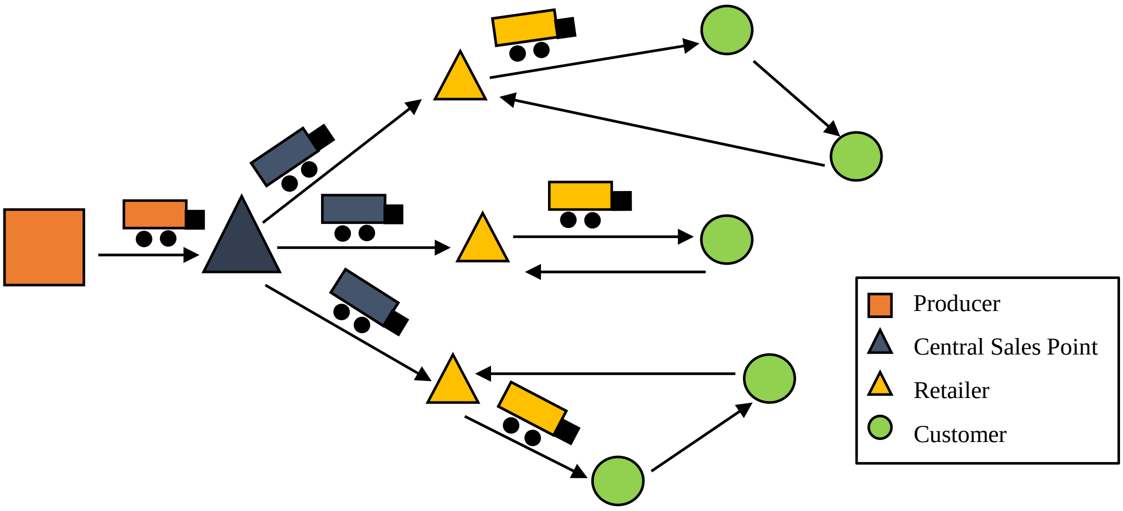
The problem considered here [Meric *et al.* 2012] originated from an industrial case of a distributor in Turkey, responsible of the distribution activities in the central part of Anatolia for a major manufacturer of furniture and home accessories. Currently the distribution and the storage activities are decentralized; the retailers are responsible for storing, displaying and delivering the products to the final customers using the trucks and the personnel owned/rented by each retailer. Once an order has been placed, the retailer, either delivers the ordered products to the customer within a service time window or transmits the order to the distributor in case of stock out, which, in turn, delivers the items to the retailers during a specific time interval. The description of the current system is illustrated in Figure 1.

The main problems are:

1. Inefficient use of retailers' transportation capacities
2. Unnecessary unloading and loading that occur when the distributor delivers the products to the retailer
3. Lack of standardized delivery procedure to the customer and lack of professionalism since the retailer delivers the items to the customer themselves with varying means and methods. Quality of product and service are not very often respected by retailers which can be evidenced by high levels of customer complains due to damaged products and unsatisfying service.

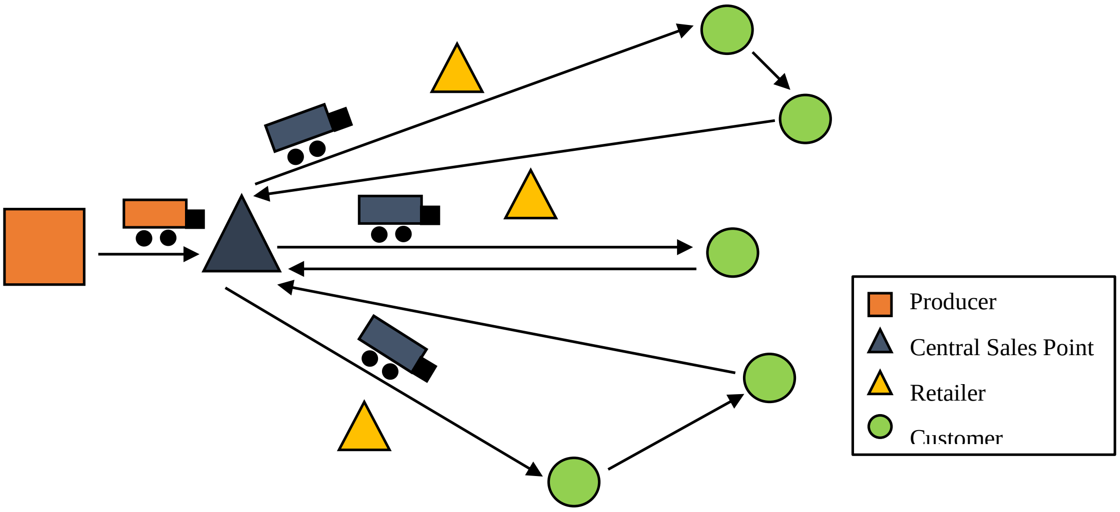
The distributor wants to establish a centralized distribution system where inventory will be kept in one warehouse and distribution activities to the final customer will be directly carried out by this warehouse. Several advantages are expected from centralization such as the standardization and better control of the distribution process, the elimination of unnecessary loading and unloading activities of the products at the retailers, the minimization of the total stock quantities in the overall system and the better utilisation of the fleet due to consolidated deliveries to the customers. Besides the advantages of the new system, there are also some disadvantages to take into account. In the current system the central warehouse can send different quantities of products to the retailers even if the latter did not place any order for those items. Once received these items, the retailer has the complete responsibility to sell them to reduce the stock level. This sale practice forces the retailers to sell always more. In the new system such forcing strategy will not be physical but will appear on the virtual environment of an integrated information system, which is less powerful. Another disadvantage can be the impossibility for customers to carry the products directly from retailer’s showrooms. Sometimes a customer can abstain from ordering a product and go to search it somewhere else because the retailer does not have it in stock.

Figure 1. Current distribution system – Decentralized distribution



In the new system, as illustrated in Figure 2, all the orders will be sent to the central warehouse who will be responsible for delivering products directly to the customers within a certain service time window in days. Retailers will only expose the products and take end-customer orders. Moreover, in this system the area served by the distributor will be divided into regions and each vehicle will be assigned to a designated region within a specific route for visiting customer. The distribution activities will be performed as follows: all ordered products will be grouped according to the region of the demand, the order date and the time constraints on total time allowed for delivery in a day. When a vehicle is fully loaded, whether its physical capacity is attained or its maximum working time per day is reached, it can be immediately sent to the region. After leaving the central dispatch point, the loaded vehicle visits the first customer, unloads the orders and installs the furniture; the vehicle travels to the second customer and the same set of steps is repeated until the vehicle is emptied. Then, the vehicle returns to the central depot and becomes available for the next day. Since we are solving a strategic level fleet sizing problem, vehicle routing is not considered and, only average distance values, average time values and average number of customers are taken into account to compute the total routing costs. Furthermore, the approximation of routing time and cost is necessary in this particular case because we do not know the exact location and demand of final customers in the future, and we only know the number of customers and their total weekly demand, in each region.

Figure 2. New distribution system – Centralized distribution



On one hand the centralization of the distribution process tends to reduce the transportation cost if more than one customer is replenished with one shipment, on the other hand, the localization of one warehouse can be found far from the end customers increasing the shipment costs. The total cost will thus depend on the demand configuration, the partition of the distribution area into regions, the allocation policy of vehicles to those regions and the time window allowed to deliver the products to the customers. Some of those criteria will be discussed in the computational results section.

The main decision to make in the case problem is to determine the type and number of vehicles to own and rent for the distributor in order to satisfy the customers demand in a given time interval and how many vehicles of each type should be assigned to the designated regions in the served area. Because of the real world constraints, the allocation of the current retailers and their customers to the regions is vital. This is mainly explained by assigning the same carriers serving the same region considering their familiarity with the addresses. Several important parameters are considered in this problem, such as the customer demand, the structure of the distribution network and the cost functions both for the investment and operating activities over the planning horizon.

3. Important assumptions and data analysis

In fleet sizing problems there are three crucial parameters impacting the choice of the decision maker: the demand to satisfy (quantity and service level), the structure of the distribution network (and the associated transportation constraints), and the cost functions both for the investment and operating activities over the given horizon. In this section we introduce important assumptions related to those parameters and we give a relevant data analysis to describe the problem in its more general case.

The main basic assumptions considered in developing the fleet-size model include:

Demand structure

- Demands are considered as low season for six months and as high season for the remaining six months; and it also varies across days in a week,
- The planning horizon is divided into discrete *time periods* in which each time period is a day. We assume that demand behaviour over the time is periodic and will not change significantly from a week to the next, though it will change

between low and high season. Since this problem involves strategic decisions and demand structure repeats during the same season, we decided to describe seasonality based on weekly aggregate customer demands. Therefore we assume two weeks, each one representing an entire season: one for low season and the other for high season,

- Since the demand picture repeats, and the model intends to minimize the total cost for a week from low season and the total cost from a week in high season, the cost for each week in a season will be the same,
- Demand magnitude, travel time and travel distances are considered to be deterministic,
- There are enough products at the central depot in each time period to meet all demands,
- All customer demands have to be met within a given time window, *TW*. The deliveries can be carried since the next day of the customer ordering date until the last *TW* days,
- Customer demand values in high season are calculated by using a fixed percentage of increase (30%, 60% and 120%) from customer demand values in low season,
- There are different families of products. In the real-life case six families are assumed; Sofa and seating, home textile, mattress, bedding furniture, furniture, and carpets.

Transportation system

- The served area is divided into nine regions,
- Fleet composition in the transportation system is heterogeneous. In this paper, we consider two types of trucks; 10 feet and 15 feet trucks,
- Owned fleet size would not change between low and high season while rented fleet size can change between seasons. We also exclude from the input owned trucks at the beginning of the planning horizon,
- Renting cost is calculated based on a profit margin from the cost of owning a vehicle. We use 3 different scenarios of profit margin in renting; 50%, 100%, and 150% of the owning cost,

- Each vehicle is only available for one period at a time and it can only make one tour at a given period due to the daily working time limitation constraint,
- We only consider the loading problem of the capacitated vehicles with items of non-identical volumes.

Note that in the modelling part of this complex problem we used many aggregated and average values such as average demands, average distances, average shopping volumes and prices, and average transportation times. Even if this aggregation step seems to be simplifier at a first glance, in such a strategic and long term decision process there is no way to use very detailed information. In order to obtain a good quality approximate solution, the use of the average values is crucial.

4. Mixed Integer Linear Problem

The mathematical model of the problem will be presented in this section with the inclusion of the above assumptions. The objective of the model is to determine the optimal fleet size and its composition while minimizing the cost of owning and renting a fleet and operational cost of deliveries based on approximate length of the routes. The fleet configuration will also depend on high and low seasonality of customers demand.

4.1 Sets and indices

I: product families, $i \in \{1...I\}$
J: set of regions, $j \in \{1...J\}$
K: type of vehicles, $k \in \{1...K\}$
T: time horizon, $t \in \{1...7\}$
U: time horizon for the deliveries, $u \in \{1...7\}$
TW: time windows (number of periods allowed for the shipment of products)
S: season, $s \in \{1, 2\}$ which represents low and high season, respectively

4.2 Parameters

d_{ijts}: customers demand in period *t* of the region *j* for the product type *i* during season *s*

sh: average shopping price of a customer; expressed in [local currency]

m_i: selling price of the product *i*; expressed in [local currency]

v_i: volume of the product *i*; expressed in [m^3]

c_k: capacity of the vehicle type *k*; expressed in [m^3]

h: daily working time limitation for a vehicle; expressed in [min]

tr: average transit time between customers; expressed in [min]

g_j: average round-trip time from the depot to the region *j*; expressed in [min]

s_i: average installation time of the product *i*; expressed in [min]

e: average distance between customers; expressed in [km]

l_j: average round-trip distance from the depot to the region *j*; expressed in [km]

p_k: unit price per km of a vehicle type *k*; expressed in [local currency/km]

f_k: weekly cost of owning a vehicle type *k* (fixed and variable costs included); expressed in [local currency]

r_k: weekly cost of renting a vehicle type *k* (with a fixed percentage of profit margin); expressed in [local currency]

4.3 Design variables

Main decision variables

W_{ijkuts}: the amount of product type *i* shipped in the vehicle type *k* to the region *j* in period *u* (with an initial ordering date in period *t*), during season *s*

X_{jkus}: the number of vehicles type *k* replenishing the region *j* in period *u* during season *s*

Y_{jks}: the number of owned vehicles type *k* assigned to the region *j* during season *s*

M_{jks}: the number of rented vehicles type *k* assigned to the region *j* during season *s*

Decision variables as a function of the main variables

R_{ks}: the number of rented vehicles type *k* during season *s*

F_k: the total owned fleet type *k*

Z_{jkus}: the number of customers in region *j* replenished by the vehicle type *k* in period *u* during season *s*

4.4 Mathematical formulation

$$Min\ Z=\sum_{s=1}^{S}\sum_{u=1}^{7}\sum_{k=1}^{K}\sum_{j=1}^{J}p_k.l_j.X_{jkus}+\sum_{s=1}^{S}\sum_{u=1}^{7}\sum_{k=1}^{K}\sum_{j=1}^{J}p_k.e.\left[Z_{jkus}-X_{jkus}\right]+\sum_{s=1}^{S}\sum_{k=1}^{K}r_k.R_{ks}+2\sum_{k=1}^{K}f_k.F_k \quad (1)$$

Subject to :

$$d_{ijts}=\begin{cases}\displaystyle\sum_{k=1}^{K}\sum_{u=t+1}^{t+tw}W_{ijktus} & t+tw\le 7\\ \displaystyle\sum_{k=1}^{K}\sum_{u=t+1|t\neq 7}^{7}W_{ijktus}+\sum_{k=1}^{K}\sum_{u=1}^{t+tw-7}W_{ijktus} & t+tw>7\end{cases} \qquad \forall(i,j,t,s) \quad (2)$$

$$c_k.X_{jkus}\ge\begin{cases}\displaystyle\sum_{i=1}^{I}\sum_{t=u-tw}^{u-1}v_i.W_{ijktus} & u-tw\ge 1\\ \displaystyle\sum_{i=1}^{I}\sum_{t=1}^{u-1|u\neq 1}v_i.W_{ijktus}+\sum_{i=1}^{I}\sum_{t=u-tw+7}^{7}v_i.W_{ijktus} & u-tw<1\end{cases} \qquad \forall(j,k,u,s) \quad (3)$$

$$Z_{jkus}=\begin{cases}\displaystyle\frac{1}{sh}.\sum_{i=1}^{I}\sum_{t=u-tw}^{u-1}m_i.W_{ijktus} & u-tw\ge 1\\ \displaystyle\frac{1}{sh}.\left(\sum_{i=1}^{I}\sum_{t=1}^{u-1|u\neq 1}m_i.W_{ijktus}+\sum_{i=1}^{I}\sum_{t=u-tw+7}^{7}m_i.W_{ijktus}\right) & u-tw<1\end{cases} \qquad \forall(j,k,u,s)\,(4)$$

$$h.X_{jkus} \geq \begin{cases} tr.\left[Z_{jkus} - X_{jkus}\right] + g_j.X_{jkus} + \sum_{i=1}^{I}\sum_{t=u-tw}^{u-1} s_i.W_{ijktus} & u - tw \geq 1 \\ tr.\left[Z_{jkus} - X_{jkus}\right] + g_j.X_{jkus} + \sum_{i=1}^{I}\sum_{t=1}^{u-1|u\neq 1} s_i.W_{ijktus} + \sum_{i=1}^{I}\sum_{t=u-tw+7}^{7} s_i.W_{ijktus} & u - tw < 1 \end{cases}$$

$$\forall(j,k,u,s) \qquad (5)$$

$$X_{jkus} \leq Y_{jks} + M_{jks} \qquad \forall(j,k,u,s) \qquad (6)$$

$$F_k \geq \sum_{j=1}^{J} Y_{jks} \qquad \forall(k,s) \qquad (7)$$

$$R_{ks} = \sum_{j=1}^{J} M_{jks} \qquad \forall(k,s) \qquad (8)$$

$$X_{jkus}, R_{ks}, F_k, W_{ijktus}, Z_{jkus} \geq 0 \qquad \forall(i,j,k,t,u,s) \qquad (9)$$

$$Y_{jks}, M_{jks} \in Z^+ \qquad \forall(j,k,u,s) \qquad (10)$$

In this formulation, the objective function (1) minimizes the total cost of routing, as well as the total cost of owning and renting a fleet during low and high season. The two first terms calculate the operational cost of deliveries based on different lengths of the routes and types of trucks; the first term determines the variable cost of utilizing a truck per kilometre travelled from the depot to each region *j* while the second term estimates the variable cost of using a truck per kilometre travelled from one customer to the next. A linear approximation is used to represent the local distance. Likewise [Redmer *et al.* 2012] use a linear function to represent the routing distance. The third term defines the cost of renting trucks for low and high season, separately. Finally, the fourth term defines the ownership cost of a fleet of trucks, which must be same in both seasons. Constraints (2) ensure the satisfaction of the customers’ demand which means that ordered products are delivered to customers within a specific service time window. Constraints (3) represent the capacity limitations of each type of truck. The aim of these constraints is to prevent trucks from overloading while assuring that transportation is carried out according to technical restrictions of the trucks. Constraints (4) estimate the number of customers in each region *j*. Given the average amount spent by a customer and the selling price of a product, we determine the average number of customers (unloading points) on a particular region for any given demand quantity. Constraints (5) ensure that the total duration of all roundtrips made on a route by each type of truck cannot exceed the total available time of the driver. The operating time of a truck (including routing time and installation activities) must be less or equal than the daily maximum working time *h.* Constraint (6) ensures that the number of trucks sent from the depot to the region *j* in each period *u* is not larger than the total number of owned and rented trucks allocated for that region. Constraint (7) guarantees the total owned fleet remains the same in both seasons. Once we decide the number of owned trucks of each type, required for a particular season, the maximum of these numbers determines the total number of purchased trucks between low and high season. Constraint (8) ensures that the number of rented trucks in season *s* is equal to the number of rented trucks allocated across all regions during the same season. This constraint also shows that rented trucks can change between seasons. Constraints (9) and (10) describe the feasibility domain of each decision variable. It should be mentioned that constraint (10) imposes the integrality requirements for variables Y_{jks} and M_{jks} and thereby makes it possible to obtain exact values for other important decision variables such as R_{ks} and F_k, which are the main focus on this paper thus determining the fleet size. Constraint (9) does not impose integrality requirements making it possible to reduce the complexity of the model hence the computation time.

A feature worth mentioning with the mathematical model is that constraints (2), (3), (4) and (5) have to be doubled in order to represent the circular ordered wrap sets from one week to the next. Because of the definition of the time horizon from 1 to 7, the model forces to treat the end of the set as wrapping around to the beginning, which becomes necessary when the time period *u* is not included on the same set of the time period *t.*

5. Computational Results

The optimization model above is a mixed integer linear programming model. We implemented this model in the optimization software Xpress-IVE using Xpress version 7.8 on a PC Lenovo IdeaPad U310 1.7 - GHz Intel Core i5-3317U processor, 4GB of RAM.

5.1 Data sets

This section shows some of the areas of application of the optimization model. Experiments are run with data obtained from the furniture and home accessories distributor such as customers demand, selling price of products, products volume and daily working time. Some other parameters are generated as realistically as possible (i.e. average routing time and distances, average installation time of products, and owning and renting costs). We describe some of those parameters in Table 2.

Table 2. Experimental Data

Parameters	Values	Units
Sh	1383.5	[local currency]
m_i	350, 350, 2000, 3250, 100, 350	[local currency]
v_i	1.2, 0.135, 2.5, 2.95, 0.15, 0.324	[m^3]
c_k	11.38, 20.75	[m^3]
H	540	[min]
Tr	15	[min]
g_j	51.2, 51.4, 41, 42, 40, 44,28, 24, 20	[min]
s_i	30, 15, 90, 165, 5, 10	[min]
E	3	[km]
l_j	25.7, 25.8, 20.5, 21, 19.9, 21.9, 13.8, 11.6, 8.1	[km]
p_k	0.9, 0.9	[local currency/km]
f_k	1322.41, 1350.41	[local currency]

In this study, two types of trucks are available to carry out distribution; 10 feet and 15 feet trucks. Other types of trucks with larger capacities such as 24.49 m^3, 28.74 m^3 and 40.15 m^3 have also been considered in the initial data sets, however, after several tests we have realized that the model typically selects trucks with smaller capacities due to the daily working time limitation.

The weekly cost of owning a truck is divided into fixed and variable costs. The first set of costs includes truck depreciation, traffic insurance, fees and taxes; and the second one includes drivers' salary, tires, maintenance and repair. The *Straight-Line Basis* method is used to calculate depreciation of the trucks. We assume the useful life of a truck as 5 years and the time value of money is also considered to calculate the salvage value of the trucks. The rest of the data is obtained from real sources in Turkey during the last two years.

To calculate the renting cost of a truck we assume that rental firms have a profit margin of 50%, 100% and 150% from the cost of owning this truck. Thus, having 1983.61 and 2025.62 for the first scenario, 2644.81 and 2700.82 for the second one, and 3306.01 and 3376.03 for the third scenario of profit margin. It should be mentioned that renting costs are expressed in local currency.

5.2 Result tables, comments

We solved various scenarios in the numerical analysis part. The scenarios are based on different levels of the following three parameters:

- The low season demand increased by 30%, 60%, and 120 % to obtain the high season demand,
- The profit margin in renting price (50%, 100%, and 150 %),
- The service time window in days (from 1 to 5 days).

Thus, the total number of scenarios is 45 (=3x3x5). All these scenarios together with the total cost obtained and the CPU times are reported in Annex A. We allowed a maximum computation time of 3600 seconds for all scenarios. The results show that 19 out of 45 scenarios are solved optimally within the maximum allowed computation time of 3600 seconds, and the average gap between lower and upper bounds across all the solutions is 1.28%. Therefore, we can carry on some analysis and derive interesting conclusions from these results for such a strategic level problem.

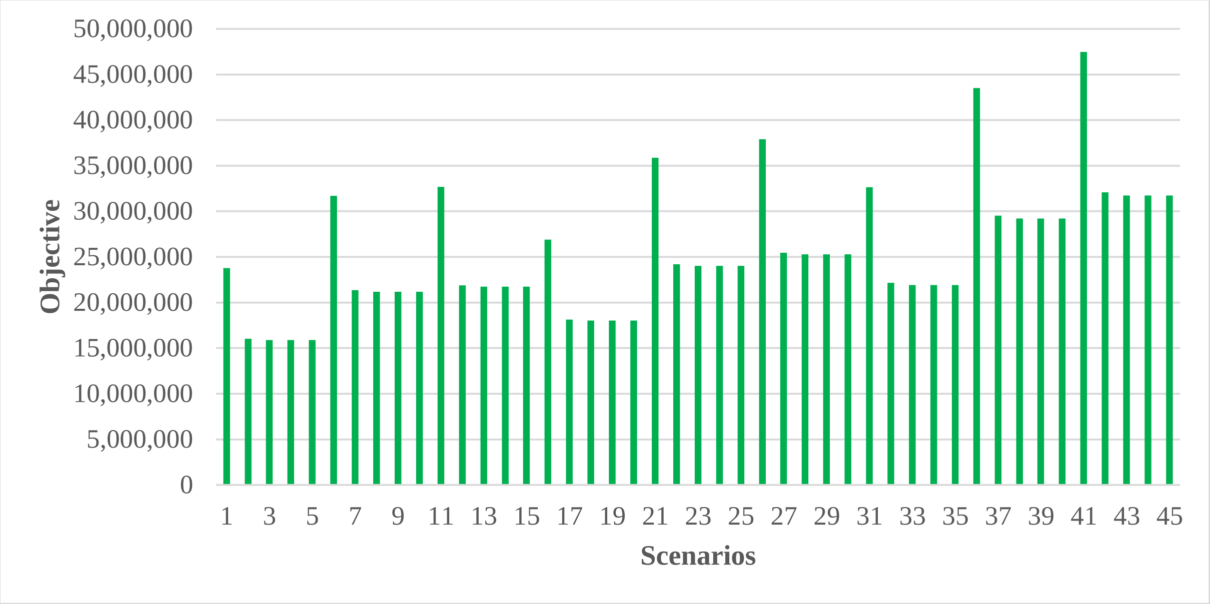
Computation time

According to the CPU time results reported in the annexe, we can see that the easiest problems are those with 1 day of service time window. The CPU time increases with the increase in service time window, as well as the complexity of the problem. This can be explained by the fact that larger service time windows allow further alternatives and mix of solutions while short service time windows severely reduce the range of solutions for the deliveries.

Objective function

Considering the total costs illustrated in Figure 3, we observe that setting service time window to 1 day significantly increases the total cost. Comparing 1 day time window scenario with larger time window scenarios, there is an increase above 33% in the total cost for the first case. Moreover, the total cost of the scenarios with service time window larger than 1 day is quite similar. The reason for the significant drop in the total cost passing from 1 to 2 days is the ability to consolidate deliveries and to better utilize the trucks. This fact is exemplified in Figure 4 and Figure 5 which give the average utilization rate of the trucks over all scenarios. There is very limited drop in total cost when we further increase the service time window due to the fact that we are unable to further consolidate the deliveries because of the time restriction on daily deliveries. Therefore, for our case problem it makes sense to set the service time window to 2 days since customers would prefer smaller service time windows.

Figure 3. Comparison of the total costs of all scenarios



Another observation we can make in Figure 3 is that there is significant cost increase when we go from 50% to 100% in the renting price profit margin while much smaller increase when we move from 100% to 150% in this parameter. We can explain these results by checking Figure 6 that illustrates the fleet configuration. Having the same scenario of demand in high season (30%, 60%, or 120%), the total fleet size stay approximately constant. However, the fleet composition differs based on the profit margin of renting price. For the cases with profit margins of 50% and 100%, we use only renting options in both seasons while when the renting price is too high (150% profit margin) we switch to owning option in low season and a mix of owning and renting in high season. That is, when we see the first increase in renting price. Even though the profit margin is settled in 100%, renting still stays as a better option and it obviously causes an increase in total cost, but when the renting cost increases further, we turn into owning option and keep the total cost more or less at the same level of those scenarios with 100% renting price profit margin.

Figure 4. Comparison of the average utilization rate of the fleet of all scenarios during low season

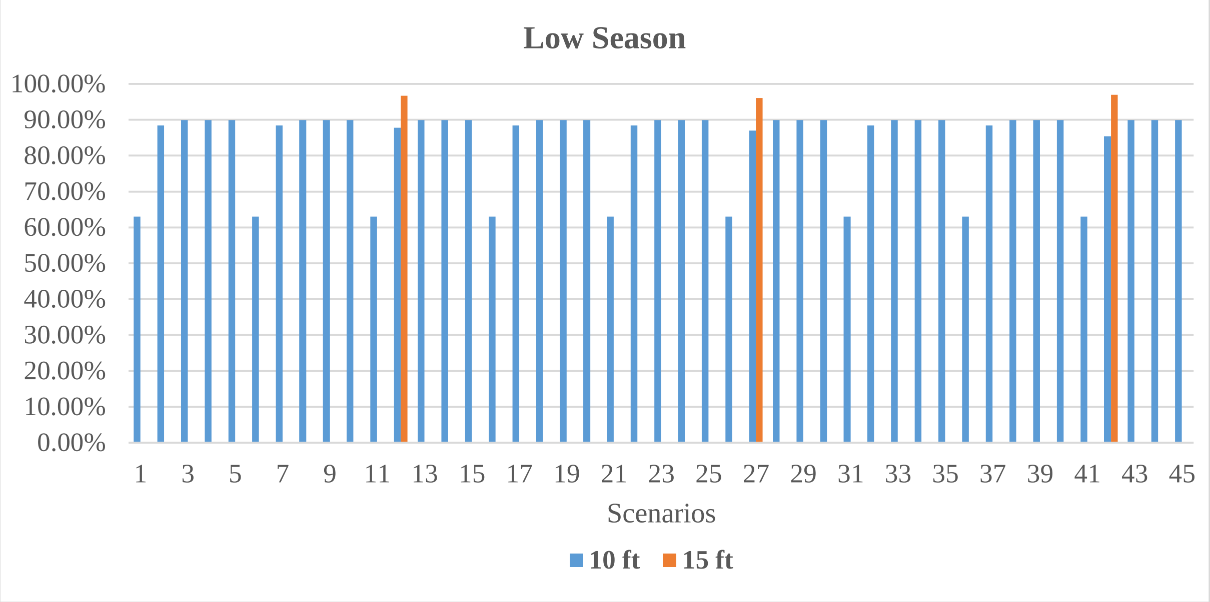
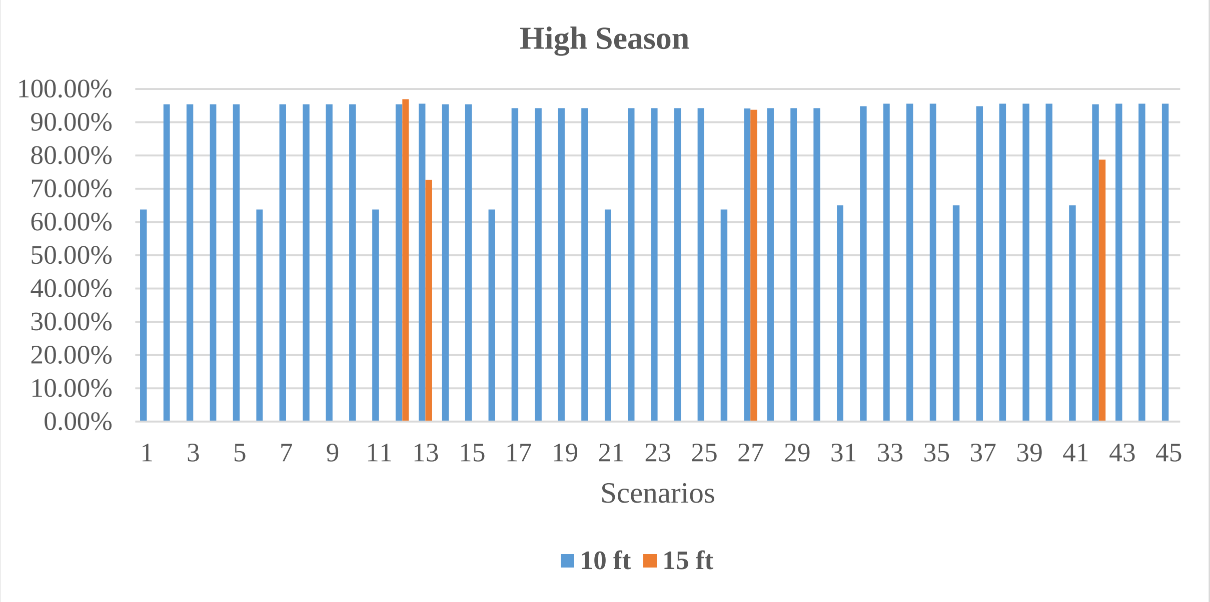


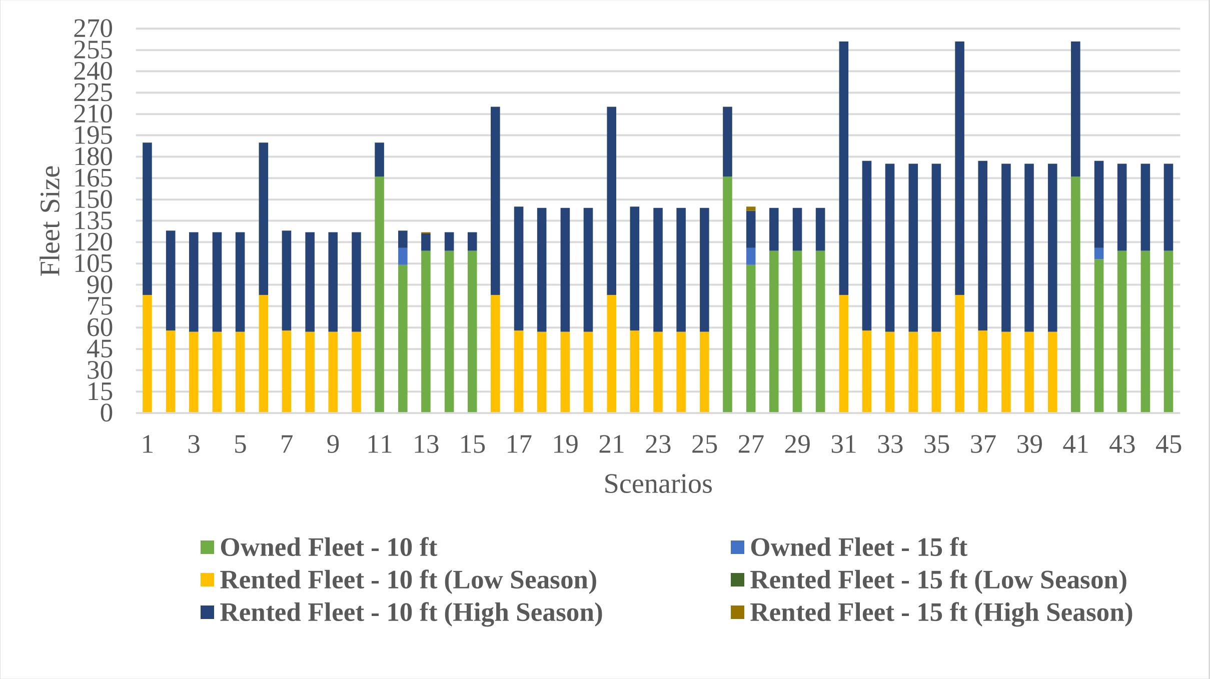
Figure 5. Comparison of the average utilization rate of the fleet of all scenarios during high season



Fleet configuration

From Figure 6 we can also see that almost in all scenarios the fleet is mainly composed of 10 feet trucks. Only in four scenarios (scenarios 12, 13, 27 and 42) we own or rent 15 feet trucks. The reason of not selecting 15 feet trucks is that the daily working time allowed for a truck to deliver products reduce the consolidation opportunity severely, thus purchasing bigger trucks is not only more expensive but useless. As we can see in the utilization chart from Figure 5, we are able to utilize the 10 feet trucks at most 95% (in high season) across all scenarios, and this explains why there could be limited need to use larger trucks.

Figure 6. Comparison of the fleet size and its configuration of all scenarios



Renting option gives us the flexibility of changing the fleet size from low to high season. The owned part of the fleet must stay the same in both seasons since owning requires us to keep the trucks during their economic lives. If the renting cost is not too high it makes sense to compose the fleet using only renting option because of the flexibility in changing fleet size. This trade-off is clearly depicted in Figure 6. For the scenarios with 50% and 100% of profit margin in renting price we exclusively use renting option, while for the scenarios with 150% profit margins in renting price the fleet is mainly composed of owned trucks with some renting to cover the peaks in demand during the high season.

According to [Imai *et al.* 2007] these results are in line with the expectations. Generally, when the fixed costs (owning or renting a fleet) and the operating costs of the fleet size that satisfies the peak demand are very high, the firm can also invest in an intermediate size of fleet between the highest and lowest demands estimated during the time horizon, and in the operational phase the firm can rent an additional fleet of vehicles to cover the excess of the demand.

Costs structure

Regarding the total cost of the solutions, we can see from Figure 7 and Figure 8 that it is mainly composed of the so called fixed (or sunk) cost of renting/owning the fleet. The operational costs, or routing costs, constitute less than 10% in all scenarios and in both seasons, which is in line with the practice in logistics. We can conclude that capital costs have a significant impact on the structure of economic activities for distribution businesses and so, making decisions at strategic level should be of great interest for those firms. Even though operational costs might seem important on a daily basis, in a long-term picture and compared to the fixed costs, the routing costs turn out to be irrelevant.

Figure 7. Comparison of the total costs of all scenarios during the low season

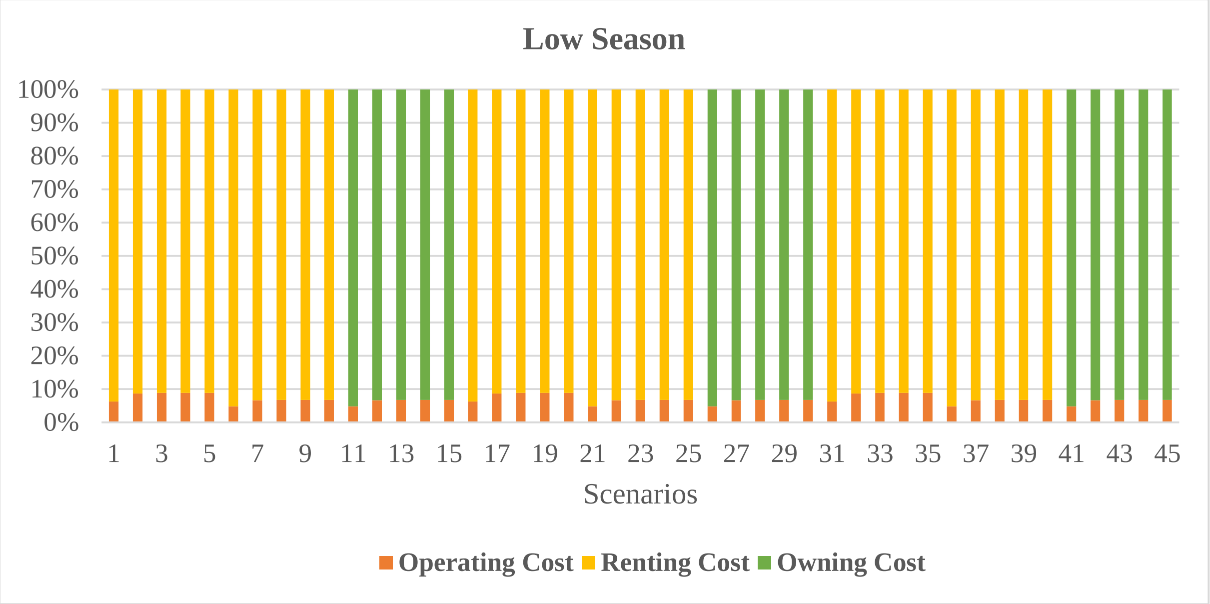
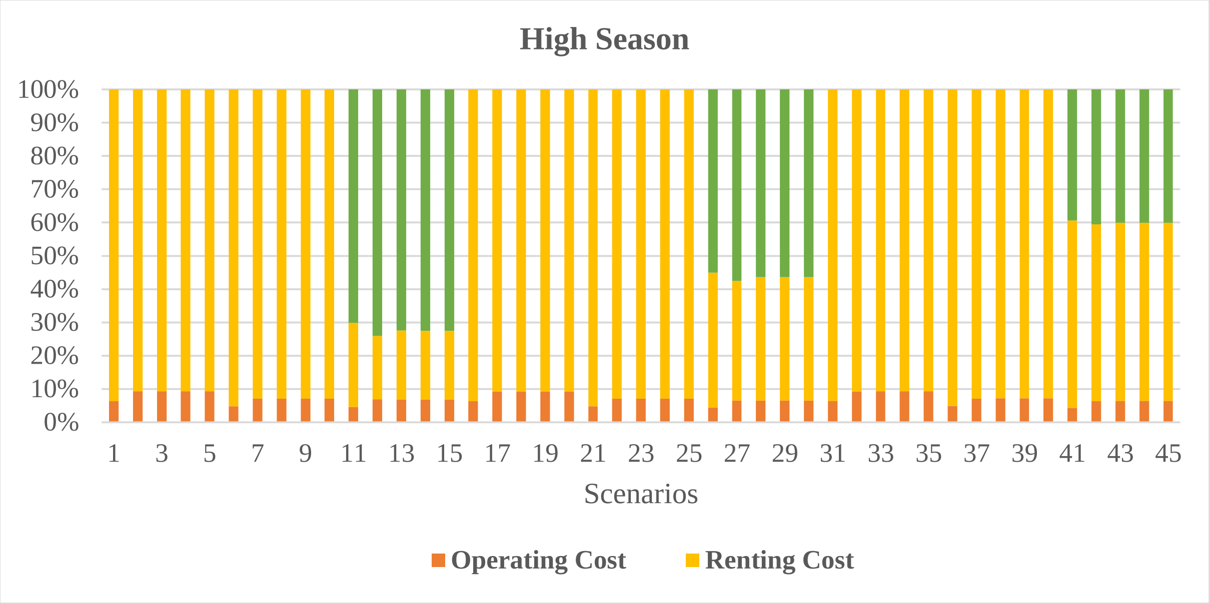


Figure 8. Comparison of the total costs of all scenarios during the high season

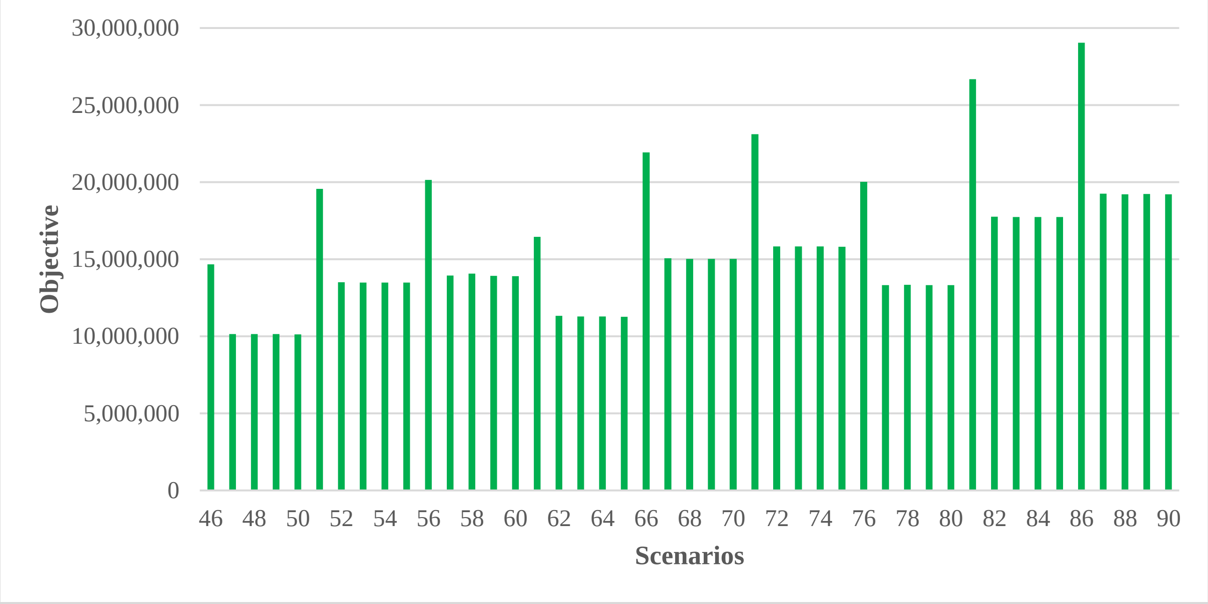


5.3 Effect of the variation in the Installation Time

In this section, we present the results of computational experiments to analyse the effects of reducing the installation time parameter on the fleet size and its configuration. For this purpose, we decrease the average installation time of each product in 50% from the original data. Results of this experiment are shown in Annex B. A common time-limit of 3600 seconds was imposed on the solution time of all the scenarios.

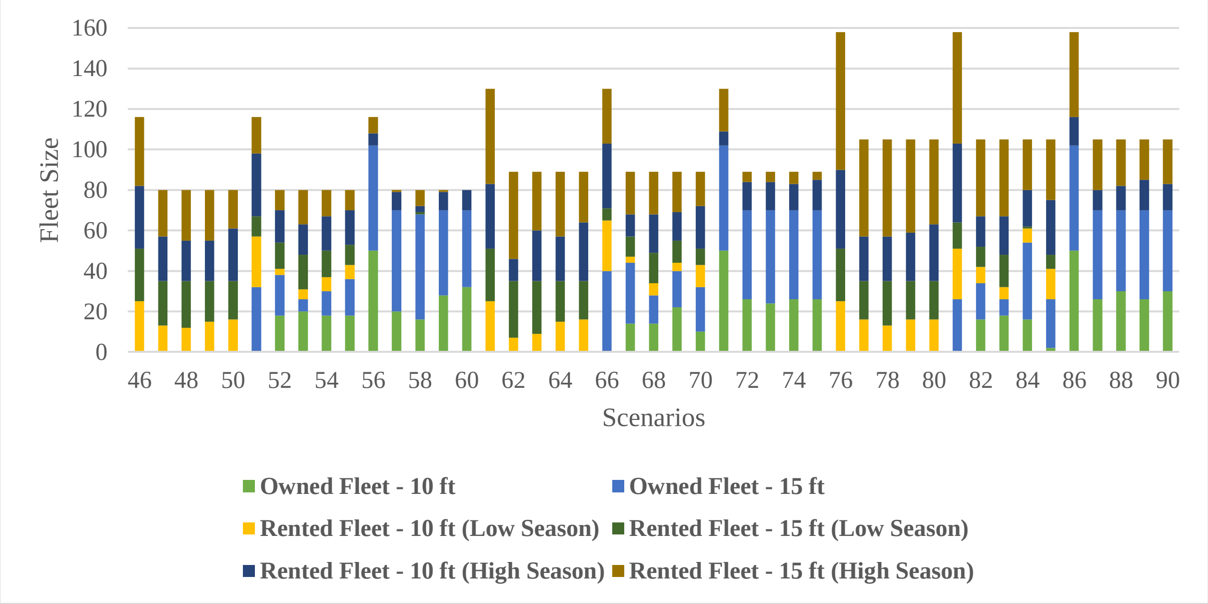
Comparing the objective for the solutions reported in Figure 9 with the ones obtained from the initial data sets, we can see a significant drop on the total cost function (about 37% less) for the same scenarios which can be easily explained by the fact that the total fleet size has also decreased, as shown in Figure 10. From the CPU time results we can also notice that the easiest problems are still those with 1 day of service time window while problems with larger service time windows are more complex, thus requiring longer CPU times. Considering the objectives values, we can say that total cost decreases from 1 to 5 days service time window, which is in line with the initial results.

Figure 9. Comparison of the total costs of all scenarios



From Figure 10 we can also see that in all scenarios the fleet is composed of 10 feet and 15 feet own and/or rent trucks. These results demonstrate that time limits have an important effect on fleet configuration thus in consolidation opportunity. By reducing the average installation time of products in 50% we are able to further consolidate the deliveries, which makes vehicles with large capacities more profitable for transportation.

Figure 10. Comparison of the fleet size and its configuration of all scenarios



Considering the average utilization rate of the trucks over all scenarios, we can see from Figure 11 and Figure 12 that we are now able to utilize the 15 feet trucks even more than the 10 feet trucks, and at most 100% in both seasons. This also explains why a diminution on the total installation time of products has a positive effect on the usage of trucks with larger capacities due to the consolidation effect of the demand and the possibility of shipping more products to customers with the same truck on a day.

Figure 11. Comparison of the average utilization rate of the fleet of all scenarios during low season

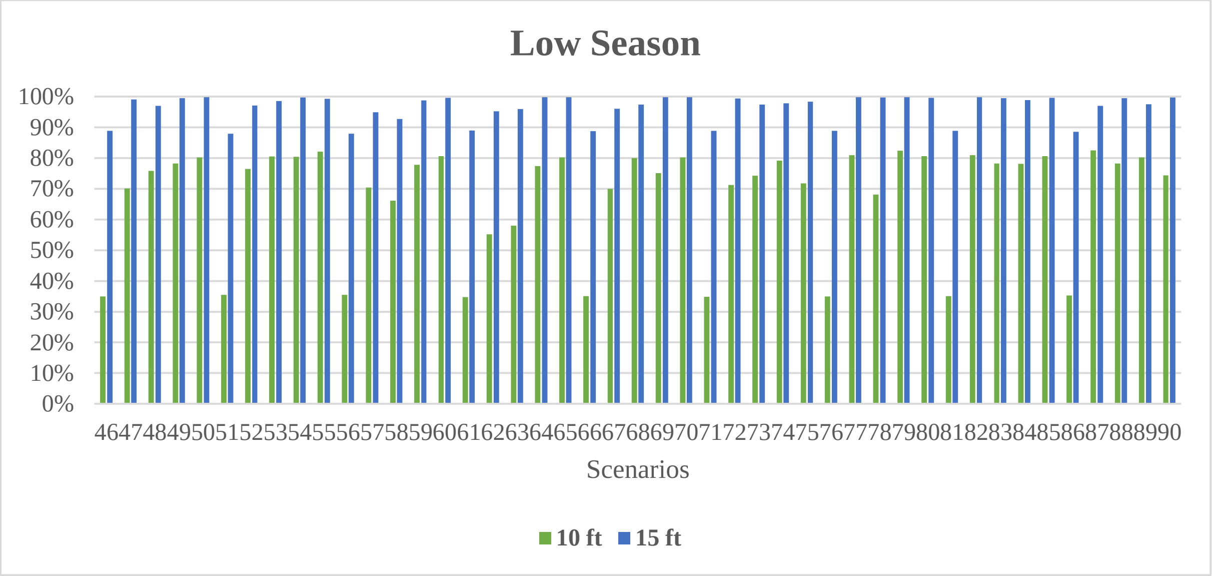
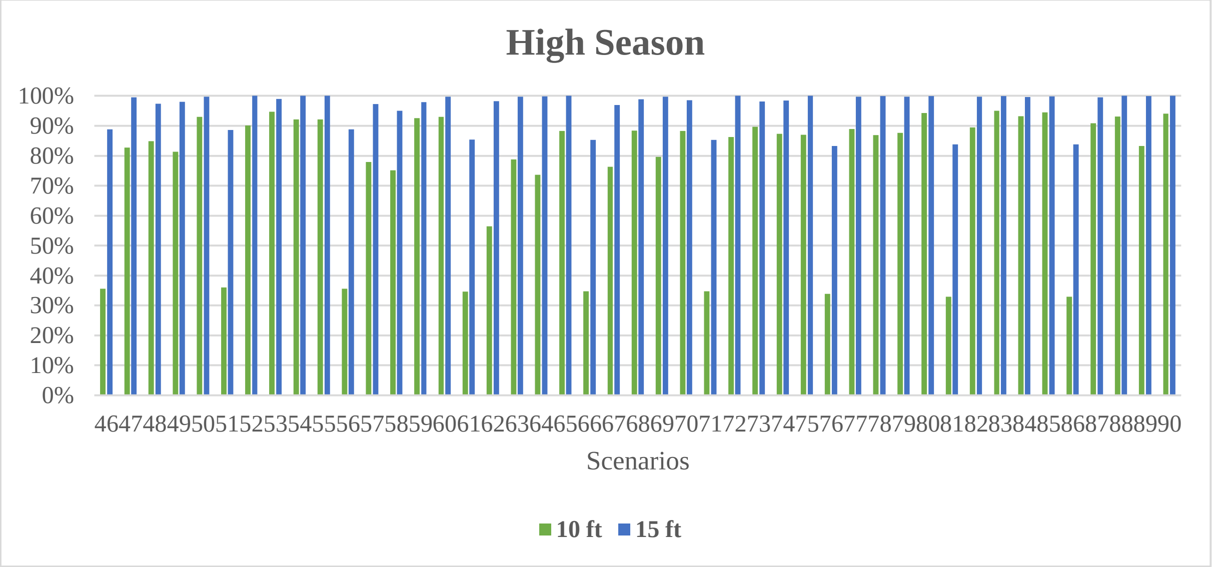


Figure 12. Comparison of the average utilization rate of the fleet of all scenarios during high season



Supplementary numerical examples

Two additional sets of scenarios were solved in order to further explore the effect of varying the installation time of products on the fleet size. For both cases we use the following levels of the main three parameters:

- The low season demand increased by 30% to obtain the high season demand,
- The profit margin in renting price of 50%,
- The service time window in days (from 1 to 5 days)

Thus having 5 scenarios (=1x1x5) for each set of experiments. Furthermore, we include two different types of trucks with larger capacities; 17 feet and 20 feet trucks, each one with 24.49 m^3 and 28.74 m^3 of volume capacity.

On the first group of scenarios, we decrease the installation time of all products in 50% from the initial data set while on the second group, we decrease the same parameter in 50% only from the product with the highest value of installation time, then the one requiring 165 minutes to be completely installed.

In both cases, the results showed that supplementary trucks are also selected for deliveries which is consistent to previous results. Even though 17 feet and 20 feet trucks are used during both seasons and in almost all the scenarios, their total number is fewer than the number of trucks of smaller capacities. This can be explained by the fact that utilizing trucks with larger capacities implies higher operational and fixed costs than those from trucks with smaller capacities, which makes the consolidation opportunity less attractive. Numerical results and some charts from both sets of scenarios are given in Annexe C and Annexe D, respectively.

Another observation we can make from these results is that the total costs have the same structure as the total costs obtained from the previous numerical experiments. It means that 1 day of service time window has the highest total cost with an increase above 32% from larger service time windows. Moreover, the CPU times reported in each Annexe are all the same, thus having a computation time of 3600 seconds for all scenarios. The main reason of getting those high CPU times is that, the inclusion of two extra types of trucks into the parameters set severely increase the complexity of the problem and the chance of mixing alternatives for the solutions.

6 CONCLUSIONS AND FURTHER WORKS

During the last decades, the importance of managing freight transportation has exponentially increased because of its high impact on the cost structure for distribution firms. Distributors are now faced with multiple constraints when making operational and strategic decisions. In addition, they have to deal with quality and service levels in order to satisfy customers' demand in required time intervals.

This study presents an optimization model for the strategic fleet sizing problem for a major furniture and home accessories distributor in Turkey, who wants to consolidate the storage and distribution activities into one warehouse. For this purpose, a literature review on the fleet sizing problem is presented in here addressing the recent studies which are close to our case. Moreover, a mixed integer linear program is proposed to determine the total number and type of owned and rented vehicles during low and high seasons, and their assignment to designated regions. If the fleet size is large, the service level is higher but the total operational and fixed costs are also high. On the contrary small fleet sizes incur on unsatisfied demands, penalty costs and low service quality. This model aims to give important insights for the optimal fleet composition at a strategic level, while taking into account average time and cost values as well as real-life vehicles' capacities and time limitations. A numerical example based on the industrial case is given to illustrate the model application. The computational results demonstrate that the optimization model is able to give good solutions in quite reasonable time and with an average gap of 1.28%. For such a strategic model built with the aggregated values, this final gap is negligible and does not prevent us from getting important conclusions and suggesting some managerial insights.

The proposed approach has a wide range character and can be applied in any type of road freight distribution firm. Two directions of further research are suggested. One of them should focus on extending the optimization model into a stochastic model to incorporate uncertainties in the model formulation. The second direction of future considerations should focus on developing heuristics-based approaches to solve larger problems with reasonable CPU times.

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ANNEXES

Annex A. Industrial case results of all scenarios.

Scenario	High Season Demand	Renting Price	Time Windows in days	CPU Time in seconds	Objective in local currency
1	30%	50%	1	0.1	23,769,000
2			2	455.5	16,020,900
3			3	3600	15,896,000
4			4	280.5	15,896,000
5			5	8.8	15,896,000
6		100%	1	0.1	31,683,600
7			2	3600	21,352,900
8			3	3600	21,186,200
9			4	3600	21,186,200
10			5	3600	21,186,200
11		150%	1	0.1	32,683,400
12			2	3600	21,895,100
13			3	3600	21,732,200
14			4	3600	21,727,800
15			5	3600	21,727,800
16	60%	50%	1	0.1	26,896,500
17			2	3600	18,148,700
18			3	459.9	18,023,700
19			4	1164.2	18,023,700
20			5	31.4	18,023,700
21		100%	1	0.1	35,852,400
22			2	3600	24,188,700
23			3	3600	24,022,100
24			4	3600	24,022,100
25			5	3600	24,022,100
26		150%	1	0.1	37,893,700
27			2	3600	25,452,400
28			3	3600	25,271,800
29			4	3600	25,271,800
30			5	3600	25,271,800
31		50%	1	0.1	32,651,600
32			2	191.3	22,154,200
33			3	3600	21,904,200

34	120%		4	17.7	21,904,200
35			5	105.2	21,904,200
36		100%	1	0.1	43,523,700
37			2	3600	29,527,200
38			3	3600	29,194,000
39			4	3600	29,194,000
40			5	3600	29,194,000
41		150%	1	0.2	47,481,100
42			2	3600	32,096,500
43			3	3600	31,735,000
44			4	3600	31,735,000
45			5	3600	31,735,000

Annex B. Results showing the effect of the variation in the installation time of products.

Scenario	High Season Demand	Renting Price	Time Windows in days	Installation Time	CPU Time in seconds	Objective in local currency
46	30%	50%	1	50%	0.2	14,675,700
47			2		3600	10,137,300
48			3		3600	10,145,200
49			4		3600	10,137,300
50			5		3600	10,118,900
51		100%	1		0.5	19,560,700
52			2		3600	13,502,500
53			3		3600	13,491,900
54			4		3600	13,484,900
55			5		3600	13,484,900
56		150%	1		0.3	20,151,000
57			2		3600	13,947,900
58			3		3600	14,073,400
59			4		3600	13,919,900
60			5		3600	13,901,400
61	60%	50%	1		0.4	16,462,300
62			2		3600	11,333,200
63			3		3600	11,290,900
64			4		3600	11,283,100
65			5		3600	11,262,000
66		100%	1		1	21,941,900
67			2		3600	15,068,000
68			3		3600	15,029,300
69			4		3600	15,025,700
70			5		3600	15,018,700
71		150%	1		1.3	23,126,800
72			2		3600	15,821,700
73			3		3600	15,828,700
74			4		3600	15,826,100
75			5		3600	15,817,200
76		50%	1		1	20,022,300
77			2		3600	13,327,600
78			3		3600	13,335,500
79			4		3600	13,322,300
80			5		3600	13,311,800
81			1		0.4	26,686,800
82			2		3600	17,757,100

83	120%	100%	3		3600	17,743,000
84			4		3600	17,750,000
85			5		3600	17,739,400
86		150%	1		0.3	29,056,600
87			2		3600	19,247,700
88			3		3600	19,224,700
89			4		3600	19,225,600
90			5		3600	19,220,300

Annex C. Results showing the effect of the variation in the installation time of all products with four types of trucks.

Scenario	High Season Demand	Renting Price	Time Windows in days	CPU Time in seconds	Objective in local currency
91	30%	50%	1	3600	14,730,000
92			2	3600	10,177,700
93			3	3600	10,135,000
94			4	3600	10,148,100
95			5	3600	10,140,800

Figure 13. Comparison of the total costs of all scenarios

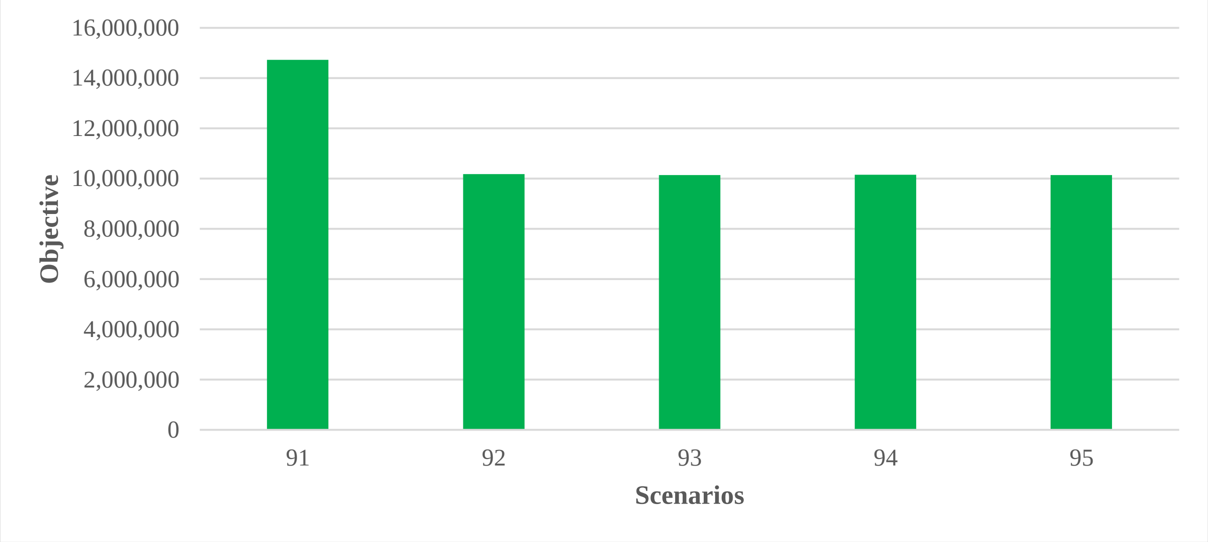
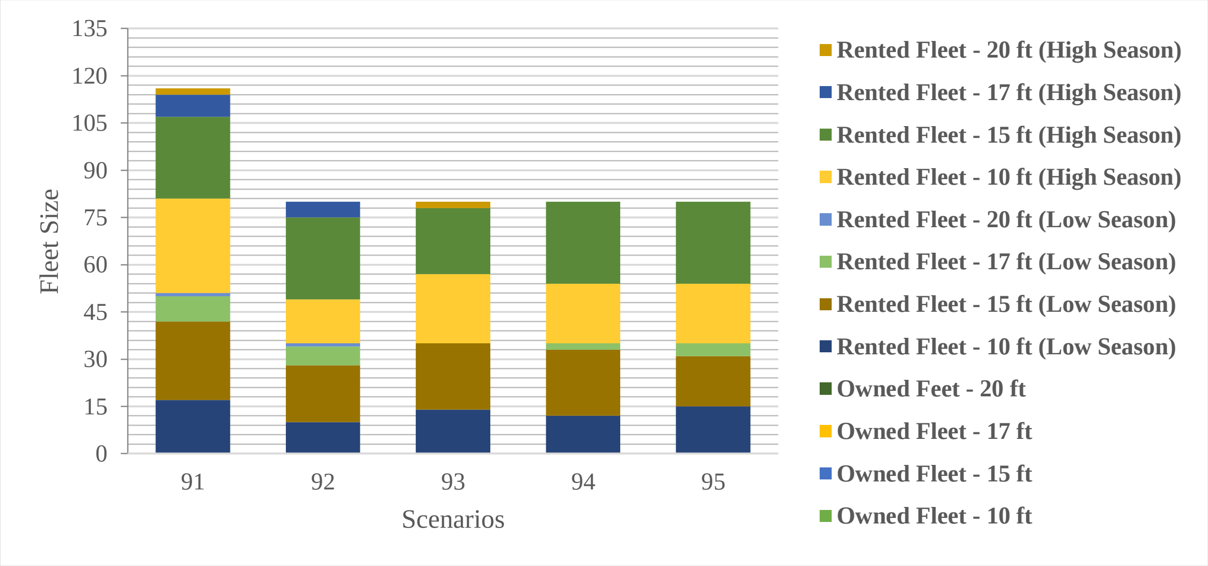


Figure 14. Comparison of the fleet size and its configuration



Annex D. Results showing the effect of the variation in the installation time of one product with four types of trucks.

Scenario	High Season Demand	Renting Price	Time Windows in days	CPU Time in seconds	Objective in local currency
96	30%	50%	1	3600	18,605,500
97			2	3600	12,800,400
98			3	3600	12,731,800
99			4	3600	12,703,700
100			5	3600	12,700,600

Figure 15. Comparison of the total costs

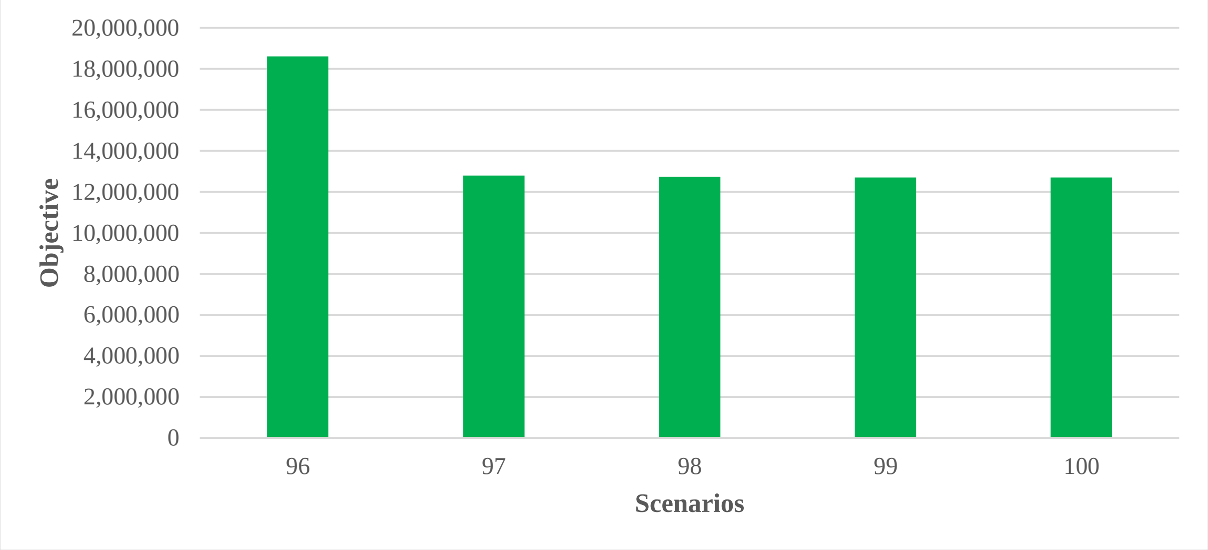


Figure 16. Comparison of the fleet size and its configuration